IPhO Lithuania 2021

IPhO 2021
Final Report

Vilnius, Lithuania
2021

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IPhO 2021 in brief numbers

Host Country	Lithuania
Way of conduction	Remote
When	Opening Ceremony on Saturday, 17 July 2021 Closing Ceremony on Saturday, 24 July 2021
Organisers	Ministry of Education, Science and Sport of the Republic of Lithuania Lithuanian Centre of Non-formal Youth Education Vilnius University
Competing Delegations	76
Students	366
Leaders	150
Observers	55
Invigilators	147
IPhO Secretariat	2
Volunteers	42
Markers	88
Awards	Gold medals – 45 Silver medals – 61 Bronze medals – 94 Honourable mentions – 62

IPhO 2021 Committee structure

Steering Committee

Steering committee was formed involving representatives of the three main organisers as well as other members who were active in the field of Physics education and science Olympiads. Steering committee made the major decisions about the Olympiad's date, form of conduction, budget, decided about the most important strategic questions and supervised the Academic, and Organising committees.

Members of the Steering committee:

* Dr. Ramūnas Skaudžius, Chairman of IPhO 2021 Steering committee
* Jonas Mickus
* Laima Rutkauskienė
* Algirdas Sakevičius
* Rigonda Skorulskienė
* Prof. Juozas Šulskus
* Dr. Paulius Lukas Tamošiūnas
* Dr. Valdas Jaskūnas
* Prof. Edmundas Kuokštis
* Justinas Baužys
* Dr. Gediminas Beresnevičius
* Justina Čiplė

Academic Committee

Academic committee consisted of Vilnius university scientists mainly. This committee was responsible for the theoretical and experimental problems, International board meetings, marking and moderation.

Members of the Academic committee:

* Prof. Edmundas Kuokštis, Chairman of IPhO 2021 Academic committee
* Prof. Andrius Alkauskas
* Prof. Egidijus Anisimovas
* Justinas Baužys
* Dr. Jevgenij Chmeliov
* Dr. Nail Garejev
* Dr. Emilis Pileckis
* Dr. Steponas Raišys
* Elvinas Ribinskas
* Dr. Mindaugas Viliūnas
* Dr. Pranciškus Vitta
* Dr. Kastytis Zubovas

Organising Committee

The Organising committee consisted mainly of the Lithuanian Centre of Non-formal Youth Education employees. This committee was responsible for registration, communication with teams, public relations, Opening and Closing ceremonies, volunteers, conduction of exams and free time activities. In the case that IPhO 2021 would have happened in the physical format this committee was also assigned the responsibility for the hospitality and logistics.
Members of the Academic committee:

- Dr. Gediminas Beresnevičius, Chairman of IPhO 2021 Organising committee
- Jurgita Brazauskaitė-Incienė
- Justina Čiplė
- Miglė Čiurinskaitė
- Žyginta Einorytė
- Liliana Feiginaitė
- Rugilė Jurevičiūtė
- Viktorija Kalaimaitė
- Vilma Maniušienė
- Liudas Mikalkevičius
- Arūnas Plikšnys
- Teklė Songulija-Tkačenko
- Milda Stachnevičiūtė
- Jurgita Stulginskienė
- Dina Voronina
- Mindaugas Žemaitis

Registration and participation issues of the delegations

The IPhO 2021 registration process started with sending official letters to the Ministries of Education of each participating country. The letters informed about the changed global circumstances due to the COVID-19 pandemic and the unknowns of the IPhO 2021 format.
According to the IPhO statutes, the official invitation letters have to be sent 6 months ahead of the event. We aimed exactly at this date to consider the latest global developments.
The official invitation letters requested a definite answer from the participants until March 15, 2021. In addition, we asked to select which format (online or physical) would be preferred for each delegation. Unfortunately, our experience shows that only about half of the delegations answered before the deadline while Kosovo and Qatar answered only in late May. Out of those who answered, 50% selected their preference as a physical format, 43.1% selected their preference as an online format, while the rest indicated that they would not be able to join due to unfavourable circumstances.
Keeping in mind that the pandemic situation was constantly changing, we allowed the delegations to edit their preference for IPhO 2021 format until May 14. As the pandemic developed, especially after the massive COVID-19 breakout in India, many countries' delegations changed their preference. By the deadline of May 14 we had to accept that the IPhO 2021 would not happen in the physical format. Therefore, on that day we sent an email explaining the situation and making the final decision to keep the **IPhO 2021 only online**.

As we had been preparing for a mixed format earlier in the year, we had already set the May 14 as a deadline to collect the shipping addresses for shipping the experiment packages. We faced major delays on this side as well. Many delegations would not provide the shipping information. In addition, those few delegations who still expected a physical format had to reorient their efforts to join the online format (which involves assigning the invigilators, finding a venue for examination as well as offering accommodation for the students, preparing to join the sessions online and so on). For these reasons we extended the deadline to provide the experiment shipping details until May 24, 2021.
Once again, we faced major challenges as only about half of the delegations submitted the shipping details by the deadline. In addition to that, we did not even know the delegation size of numerous participating countries. The deadline to provide the delegation size was April 2, but in late May we still lacked information from many delegations and those who provided their information were often requesting to change

it.
The problems stated above highly impacted the shipping process which would have been much smoother had we known the delegation size and shipping details by the deadlines we set.
The last important deadline was June 17 by which the delegations were asked to pay the fees. Most of the delegations conformed to this deadline. There were some irregularities since transferring money between some countries is a difficult challenge. There were also some delegations who failed to plan properly and missed the deadline.

Preparation for the Exams

Choosing the communications platforms

For the effective communications we needed to choose a suitable communications platform which would offer the needed functionalities and would work in all participating countries. Within the organizers two favourites were identified: Zoom and Microsoft Teams.
In order to decide and offer a democratic process, a survey was launched. The survey had the following questions:

- Your country
- Your full name
- Your role in IPhO 2021
  - Team leader
  - Observer
  - Invigilator
- Can you access YouTube freely in your country
- What messaging services are NOT available in your country?
  - Facebook Messenger
  - WhatsApp
  - Signal
  - Telegram
  - All of the above are available
- Which one is your preferred video communication platform
  - Zoom
  - MS Teams
  - Both are fine for me
  - I can't access either
- Statement: Please answer the following questions only if you have any restrictions to access the above-mentioned communication platforms.
- If you cannot access neither Zoom, nor MS Teams, have you tried using a VPN?
- What VPN provider works in your country?
  - ExpressVPN
  - NordVPN
  - PureVPN
  - Cyberghost
  - Surfshark

- If there is no possibility to use a VPN, what alternative video communication service works in your country (comparable to Zoom or MS Teams)?
- Other comments

The first part of the survey was made to understand who is answering and to make sure that the survey responder is really related to the IPhO 2021.

The second part of the survey was created to decide which video communications platform is the most suitable. In addition, we asked whether Youtube was accessible in all participating countries. Last, we asked about the social communications service.

The third part of the survey was only directed at those who faced extraordinary challenges to join the most common communication services.

We considered the possibility of providing a VPN service for those delegations who could not access Zoom, MS Teams, or Youtube. However, we later noticed that many participating countries managed to access some other international events which were organized on Zoom. We therefore decided to let the delegations solve this problem by themselves.

The survey results showed that over 95% of responders could access YouTube in their countries. In addition, only less than 3% of responders showed a preference for the MS Teams, while 44.9% showed a preference for Zoom and 50.5% stated that they could work with either platform. Based on this survey we made the following decisions:

- **Youtube** would be used for the opening and closing ceremonies
- **Zoom** would be used for all online meetings, surveillance and free-time activities

In addition to the above choices the Steering Committee decided to use **OlyExams** for the academic part of the olympiad.

The Organizing Committee decided to use the **CinemaClub** platform for viewing a Lithuanian movie which was offered as a part of the free-time activities. This choice was made because the organizers had a contact with the CinemaClub management team who graciously offered to open the platform in all participating countries.

Online invigilation

When considering the online format of the IPhO 2021 we evaluated several approaches to oversee the organization processes and keep the high transparency standards.

1. Sending the volunteers from Lithuania to the participating countries. A similar approach was used by the Russian Federation while organizing the IPhO alternative in 2020;
2. Searching for the local people in the participating countries to be volunteers and to supervise the organization of the event. The people most probably would be connected to Lithuania in some ways;
3. Asking the participating delegations to pick their invigilators (an invigilator is a person supervising the organization process and ensuring the transparency of the IPhO).

While we considered the advantages and disadvantages of all the options, we decided to stick with the third option for a few reasons.

Sending our volunteers would be a very complicated process because of the COVID-19 restrictions to travel, lack of flights to some destinations, unclear and in most cases lengthy quarantine requirements upon arrival, and the risks of getting the volunteer sick while traveling.

Secondly, searching for local people to help in supervising the IPhO 2021 was the most complicated option. While the IPhO 2021 organisers could have taken this task as a responsibility, it would be very difficult to reach people and groups in all the participating countries. Also, the selected people might have trouble working with Physics-related topics or in general might find it difficult to work in the IPhO context even if they could travel inside the country to be in the required location.

While considering the options 1 and 2 we also assumed that different delegations will have varying needs for the invigilators, the number of people, the workload, etc. **Therefore, we chose to ask the delegations to pick their invigilators to ensure the best collaboration and manage the expectations of the delegations.**

Setting the fraud prevention measures

When considering the switch from the IPhO 2021 physical format to the IPhO 2021 online format we really wanted to maintain a proper level of transparency and fraud prevention. We were organizing the proper IPhO, not a place-holder event. Therefore the stakes were very high.

After many discussions within the Organizing Committee as well as between all the committees, we came to the following list of measures to be used:

- The experiments should be shipped time-wise as close to the Olympiad as possible;

- The experiments were packaged in smaller sealed boxes within a larger shipping box;
- We used special sealing tape for the experiments;
- The experiment was designed in such a way that without the instructions and tasks, nobody could understand what exactly is to be done;
- The instructions about preparing and testing the experiments were sent to the invigilators on the night before the experiment when the team leaders were already translating the tasks;
- The invigilators were asked to keep the students away from the team leaders over a physical distance;
- All the students were supposed to stay in a hotel or a comparable facility while not taking the exams so that the location of the students could be monitored at all times;
- In the special cases where the students were unable to live at a hotel or other comparable facility, they were allowed to stay at home. However, it was still mandatory to take the exams from a specially adapted non-residential facility;
- The invigilators could not be the participating student, nor the team leader who was appointed to translate and discuss the exams. The invigilator also could not be a relative of a student, nor a friend or a colleague;
- The invigilators were responsible to know the location of the students 24/7 throughout the examination time from July 18 to July 21, 2021. The invigilators had to collect all the communication devices from the students during this time;
- The invigilators had to open the experiment kits only under surveillance of the organisers so that the organisers could inspect if the seal is intact. In the rare cases when the experiments were opened by the customs, we asked the invigilators to reseal the packages in a tamper-proof way;
- During the exams all students had to be monitored at all times. There were supposed to be cameras to film every student as well as one camera to survey the room from the back. In addition, it was recommended to have an off-line camera with a memory to record the process in case there were some internet connectivity problems;
- The invigilators were under surveillance while printing the exams, also while scanning the answers;
- The invigilators were asked to write the exam reports based on a template to indicate any unusual activities such as network interruptions, blackouts, as well as toilet breaks.

Noted breaches

- Many invigilators opened the experiment kits without a permission to open. During the Zoom session, a member of the academic committee was explaining what to do with the experiments and a lot of invigilators started opening the kits.
- There were some severe electricity and internet connectivity problems in some countries.
- Several delegations refused to use individual cameras for students quoting extreme poverty in their countries.
- We recorded invigilator confusion about the rules and intentions to reveal the experiment exam kits to the students the night before the experiment.
- Some students were discovered to have access to the mobile phones during the time when all the connectivity devices should have been collected from them.
- Some invigilators did not provide the exam reports.

Experiment shipping

Experiment shipping was an enormous logistical challenge. While it can be considered a success we will provide some bullet points about what worked and what didn't after briefly explaining the shipping process.

As one of the organizing institutions was a budget institution, the shipping services had to be obtained through a public procurement procedure. This made things more complicated but eventually a shipping partner was selected who would provide the whole shipping service through TNT, FedEx and some other rapid logistics companies.

A form was launched to collect the shipping addresses of all the delegations. The experiments were supposed to be delivered to the invigilators. Some respondents provided faulty, incomplete or mismatched addresses which made the shipping process very tedious.

In addition, all the experiment shipments had to be insured against loss or damage. Therefore, the real values of the kits were declared in the shipping documents which made the shipping invoices of about 1000 EUR. As organizers, we had to provide detailed lists of contents including the smallest items like erasers, their values, materials, manufacturers, etc.

The customs export procedures were taken care of by the assigned shipping company and after passing through the customs the packages were delivered in less than a weeks' time to the customs of the receiving end.

Our learnings

- We never managed to ship items to the Russian Federation due to the most complex customs procedures. In this case we had to ask for help from the Ministry of the Foreign Affairs of the Republic of Lithuania to organise a diplomatic mail transfer.
- Some other countries had very long but doable customs procedures. Therefore, if shipping from within the EU to the 3rd countries we suggest adding about 1-2 months extra time for the customs procedures.
- Some countries have insanely large import taxes or non-transparent customs procedures which leads to astronomical import duties. We recommend considering all options and discussing with the receivers about the best way for the imports.
- We suggest collecting the addresses very early in advance and verify with the shipping company to avoid the delays.
- We also suggest, if possible, adding some extra spare supplies (like connecting wires, switches, etc.) in a separate box for Invigilators only. In such a case, the presence of spare parts will not confuse students when they compare their experimental setup with the description given in the problem text, but the Invigilator would still have the possibility for a quick fix if some minor parts of the setup are broken or not working.
- In the end, we still had to have a plan B for the cases when the experiments did not arrive or arrived faulty. This had to be carefully communicated as to avoid the incentives to abandon the experiments and go for plan B.

Experiment related secrecy and instructions for the invigilators

As it was mentioned in one of the earlier sections, the experiment equipment was not revealed to the invigilators until the last night before the experimental exam. The exam tasks were only known to the team leaders who were supposed to be separated from the students and invigilators In the preceding communications we informed the delegations that they would need a thermometer to measure the ambient temperature as well as an electric socket for each student.
The night before the experiment the invigilators received the instructions about testing the experiments and preparing them for the exam.
It was imperative to check the tablet, charge it and reset the memory while installing the latest application version for the experiment.
The application has a testing option for which the invigilators had to assemble the chip with the provided wires and test the experiment.

The biggest obstacles we encountered were that some invigilators did not know how to work with a tablet, or did not know how to reset and install the app which was not on the app store. Also, for some it was difficult to distinguish between the new app version and the old app version (instructions were provided).
Some invigilators did not show up at the Zoom session dedicated to the opening and testing of the experiments which became an obstacle the next day when they had to do a complex task very quickly.
Some invigilators also forgot to charge the tablets which led to pausing the exam for some students while waiting for the tablets to charge (about 3 hours).

Online support during the examinations

During the experimental exam technical support groups were working to help the students whose experiments were not working as described in the experimental kit description. Technical support group was led by an academic committee member who designed an experimental app.
In addition, there were also comparatively high numbers of calls to the tech-support Zoom room: some students were not able to start the measurements correctly, the electronic board was not responding as described in the manual, the app was frozen, not updating the graph in live setting or crashing. The absolute majority of these problems were resolved with the help of our tech-support team. The students were instructed how to reset the electronic board, reload the app or the tablet, and it seemed to solve the problem (at least the students said so). Some students reported that their app was crashing continuously – our representatives were not able to reproduce that effect locally, but finally advised the students not to save the already completed measurements; as a result, the students were not able to simultaneously draw and compare several curves, but at least were able to perform the measurements and compare curves by sequentially drawing them on the paper. Some students initially had some problems with the provided blue wires – encouragement to reset/restart the measurement and try to connect pins once more seemed to have helped. One student managed to break the wire, but his invigilator succeeded in repairing it.
We must admit that in some countries, the Invigilators had not done their best effort and did not do what they were asked in the numerous emails before the Olympiad and the detailed instructions given on Sunday morning when the experimental kits should have been opened. The app in the tablet should have been updated to a new version, some basic test measurements had to be performed in order to ensure that everything was operating as expected, and the tablets had to be recharged to 100%. Exactly for this we had a support line open all Sunday and had helped quite a large number of delegations that initially had some problems in doing those steps themselves according to the provided instructions. However, for some teams the app was not

updated. One student had to take a remote exam because the Invigilator had accidentally thrown away the blue connecting wires to the trash. Some tablets were not charged before exams, so the experimental exam was postponed for these students. Other countries had technical issues regarding printing, internet connection and other problems and started their examinations later than planned. All these technical issues were documented in the exam reports and notes of the volunteers who were supervising these countries over Zoom.

Printing and scanning

Due to the online participation, the main tasks of printing, scanning and delivering the exam problems to the students were delegated to participating countries. Each country arranged their own printer(s) and scanners. Invigilators printed the experiment and theoretical problems and put them on the student tables. After the examinations they gathered the answer sheets with the worksheets, scanned them, and sent them to organisers using the OlyExams tool.

On the organisers side only two tasks were left: print the uploaded exam solutions for the Markers and scan corrections. For this task a separate room was arranged, four printers were rented from the company Konica Minolta.

Printers Bizhub 550i, automatic feeders with scanning. Main parameter for choosing a printer was the printing speed A4/A3: 55/27 ppm. Economically from budget saving perspectives is more useful to choose less speed, and choose more printers.

Rent contract included extra costs for each page over the agreed limit, but that limit wasn't reached. In the agreement there were conditions for extra emergency support from the company. It was agreed that the printers would be serviced within one hour from the call. We also booked a dedicated employee from the Vilnius University IT department who was always with the organisers during the Olympiad and was responsible for smooth flow of the printing and scanning process.

Three printers were used, one was left for emergency situations. Each printer has been assigned a 3 person crew. All printed documents were put in envelopes and given to the Markers who were sitting in the rooms near the printing office.

The organisers had no specific issues delivering the final confirmed Olympiad tasks and solutions to the Markers, because after they were confirmed we had all night to print them. And it was easier because there was no separation by countries or students.

Main complaint from the Markers was about the time schedule to receive the student solutions. We had promised them that after 1-2 hours after their assigned country finished Olympiad tasks the Markers would receive the printed answers. Due to the delayed (30-40 min.) information from countries there was a bottleneck in the printing room. We received a lot of answers at the same time, and due to this the Markers had to wait for their printed sheets for 1-3 hours.

Each Marker was assigned to grade 5 countries' solutions. There were a lot of situations

where only 3 delegations had sent in their solutions and they were printed on time. The Markers, however, had to wait for the rest of the solutions for a few hours. We strongly recommend the future organisers to prolong this time, and create a separate printing schedule to reduce this waiting time. One printer team needs to print all the solutions of the 5 countries assigned for one Marker. Also it would be nice to assign an additional printer for the Markers so they could print missing countries solutions by themselves.
There were no complaints from the Markers about the scan or printing quality, from participating countries. There were a few issues of some missing pages in the scans or mixed students in the OlyExams system. Our help centre had a lot of work to explain all procedures to the participating delegations. Most of the questions and issues were attributed to computer and printing literacy gaps. For the future organisers we strongly recommend to strictly ask that an invigilator should be a person with a good computer and internet knowledge and who knows the situation in the building where students will work with tasks.
We also suggest exploring ideas to not print the solutions and answer sheets for the Markers, and do the evaluation process fully online. About 20% of the Markers were working remotely and everything was fine. There are a lot of tools to correct scanned PDF files. Younger evaluators were keen to pursue a paperless evaluation.

Remote experiment exams for those who did not receive the shipments

The experimental task was designed in such a way that all the measurements and data collection were controlled through the application installed on the Android-based tablet that was connected to the experimental board. After experiment kits were sent to the participating countries, we still had a significant number of backup experimental kits at our disposal. This provided us with two possibilities for risk management.
First, we offered the participating countries to order an extra kit to be sent for them separately so that it could be used by a student in case of a malfunction of his/her setup (though such a possibility was minimal as we had thoroughly tested each board and tablet before sending them).
Second, we also considered the possibility to conduct the experiment remotely in case some countries did not receive their experimental kits on time. For that, we locally prepared some experimental kits and additionally installed the Remote Control application (TeamViewer) on their tablets. At their examination sites, students could use their tablets or computers (provided by their Invigilator) with the installed TeamViewer applications to remotely access the experiment tablet prepared at our site in Vilnius University. As a result, students could see the screen of our local tablet with the IPhO 2021 application running and remotely control it. Additionally, a web camera was placed over each electronic board for the live-streaming. The students

were also able to see their assigned electronic board. For the first experiment, some manipulations with the electronic board were needed (connecting some wires, switching the on-board switch)–that would be done by our Volunteers, as instructed by the students via chat or a Zoom call. The second experiment could be done completely remotely without any local intervention.
As it turned out, we indeed had to implement the second scenario for some students. During the first International Board meeting we informed the International Board that some countries had not received their experimental kits and described to them the possibility to conduct experiment remotely. It was voted that such students were given extra 30 minutes to account for the time spent on instructing our local Volunteer how to control the electronic board.

Zoom related setup and issues

2 Zoom accounts with a large meeting add-on (up to 1000 participants) and a few smaller business accounts were used for the Olympiad. 1 account with a large add-on was designated for the exams and students' leisure activities while the other one was used for the International board meetings and team leaders'/observers' free time activities. The same Zoom meeting links and IDs were used for multiple activities to avoid confusion. All countries managed to connect to the Olympiad's Zoom sessions. Some used VPNs or managed in other ways to connect.
The International board meeting was held in a designated breakout room, to which volunteers assigned only those International board members and observers, whose identity was checked according to IPhO 2021 registration. Some issues occurred, when team leaders tried to connect to this meeting with students' names, but these issues were successfully resolved.
For the exams we created separate breakout rooms with 3-5 countries' delegations and the assigned volunteers per room. Each country had 6 cameras on average. Remote experimental exam (only for those delegations who did not receive their experimental exam kits) was held in a different Zoom session with a breakout room for each student and their experiment.
The main Zoom related issue happened during the experimental exam (first of the two exams). The 'assign for breakout rooms' function crashed after reaching about 500 participants limit in the breakout rooms. Some countries have already started their examinations while others were just joining the meeting for the preparations when it happened. We created a new Zoom session for the delegations who were starting later and asked them to join the new session. Those countries who had already started their examination were left in the first Zoom meeting (the malfunctioning one). It was really messy due to some of their cameras being in breakout rooms and some in the main session. Nevertheless, with additional care of the volunteers and organisers everything was monitored and controlled. To avoid this problem the theoretical exam was conducted over 2 zoom links.

International Board Meetings online

International Board (IB) meetings were organized in a designated breakout room on Zoom. The volunteers assigned to the room only those International board members and observers, whose identity was checked according to IPhO 2021 registration. Since mostly everyone already had some experience with Zoom at their home institution, everything went very smoothly. All the problems of the Experimental (1st IB meeting) and the Theoretical (2nd IB meeting) exams were presented at once one after another. Then the team leaders were given 1 hour for each problem to provide their feedback (for that, OlyExams platform was used). During the 1st IB meeting we also had a separate Zoom room where the team leaders could connect and see the experimental setup–the electronic board, the tablet mobile application, the way the measurements should be done, etc.

All the discussions were held over Zoom, the votes and feedback were organized via OlyExams. In addition, we had a separate Tech-Support Zoom room, where leaders could connect to report any technical issues they had faced (connection issues, operation of OlyExams, translations, etc.) and get the corresponding support to solve these issues.

OlyExams coordination for Online exam system

The organization of online Olympiad was greatly facilitated with the OlyExams platform. Many leaders were already familiar with this online system as it was first introduced during the IPhO 2016 in Switzerland. This system was used to:

- Organize all votes during the International Board meetings;
- Facilitate the translation of the Exam text into the students' native language and prepare the merged problem PDF file per problem for each student (problem text in the native and/or English language + answer sheets + working sheets);
- Access the problem text by the Invigilators: they could download the generated problem PDF files for each system and then print them using their local printing facilities;
- Upload the scanned students' solutions by the Invigilators;
- Access the scanned students' solutions by our Markers;
- Enter the students' marks by our Markers and the Team Leaders;
- Facilitate the Moderation process;
- Export the student's marks for a further statistical analysis;
- Export the final Official and translated problems' text typeset in LaTeX.

Issues and matters faced during translations

Exam translation was organised using the OlyExams platform. About 10 days before the Olympiad, team leaders received their usernames, passwords, and links to connect to the Demo-version of the OlyExams platform, where they were able to familiarise themselves with this system by translating the General Instructions for the Theoretical and Experimental exams. Any issues that were faced at this stage were resolved by the tech-support team over email. Later, the translations of the General Instructions were copied to the OlyExams server (non-Demo) that was used during the Olympiad.
Thanks to this training, during the actual translation of the Exams, even the new team leaders already had some basic experience with the OlyExams system. Some technical issues appeared occasionally (mainly due to the usage of incorrect LaTeX symbols) and were resolved operatively by the tech-support team available at a separate Zoom room.
On the first Examination day, the Invigilators in some countries had to delay the printing of their problem text because they had not received them yet. It appeared that the corresponding team leaders had chosen to use the original English version (their students did not need any translations), but did not indicate that in the OlyExams system. Multiple reminders not to forget to make a final submission in OlyExams helped to avoid such mistakes during the second translation day.
There were no critical issues in this field, some country representatives forgot their logins, and login links. Processes were well defined but some steps were skipped by them due to lack of attention. We strongly recommend to assign a mentor for any new delegation to reduce their stress levels. The mentors could be representatives from other participating countries and/or dedicated volunteers.

Online Moderation experience

Online moderation was held using the Zoom platform. The country representatives and the moderators joined one Zoom channel for the ID verification. From there the volunteers assigned them to their moderation rooms. For each moderation 30 min. time windows were dedicated.
12 Zoom channels were created, with almost 40 moderation rooms. Moderation schedule was created according to the time zones of the participating countries. Some moderation sessions started at 19-22 (7PM-10PM) o'clock Lithuanian time. It was very difficult to find time for the South and North America's representatives due to the working hours of moderators. Also for some countries there were less than four hours to prepare for the appeal after medal ranges were presented.
88 Markers participated in the moderation process. Unfortunately, there were situations when the Markers did not show up, or informed us last minute that they could

not participate due to other unexpected arrangements. The organisers had an emergency task to find other moderators, and/or reschedule so that the new moderators would have time to familiarise with the solutions. We strongly recommend having 2-6 additional free moderators who are familiar with the solutions and dedicate extra time in the Moderation schedule for such cases. We were lucky to solve this problem easily with the authors of exam problems and some PhD students who were free.

Each moderation session ended with a verbal conclusion on marks. Marks were corrected by the moderators in the OlyExams system and confirmed afterwards by the country representatives.

Some country representatives did not show up for moderation and did not inform that they would not appeal. Unnecessary work was done by the volunteers trying to contact those country representatives only to find out they would not appeal.

There were also some unexpected situations. Some delegations wanted to appeal the same problems and issues a second and a third time. They did not want to move on with a three-member international board. We strongly recommend that for such cases a senior IPhO board member would be available during moderations to avoid complicated situations.

There were some technical issues with the Zoom platform and its rules. For example there is a limit on how many channels one can create per account and how many rooms one can establish. During the test everything worked perfectly, because not all of them were open at one time. 1 hour before the moderation started, we saw the problem and had to create new links for moderation from more Zoom accounts. If Zoom is used again in the future we strongly recommend having additional backup accounts.

Also it would be useful to have a separate channel to discuss marking issues with other moderators and authors during moderations, because there were a lot of questions on how to evaluate some other possible solutions and mistakes in the first step of solution. Usually, the country representatives had strong arguments for that. Also it would be useful to establish a hotline with one or two senior scientists or authors to consult moderators for strengthening their confidence in their decisions.

Three-member international board's exam handling issues online

During the moderation, a separate Zoom meeting was organised for communication between the 3 assigned IB members and the team leaders to make a decision on the final marks when no agreement between the team leaders and the markers was achieved. A representative of the Academic committee also participated in this meeting (with no voting right) to inform the 3 IB members about meeting requests from

the team leaders and also to provide the 3 IB members with the copies of the solutions of the corresponding students in question. Just several team leaders applied for this meeting, most of the questions were resolved in favour of the students.
From the organisational point, initially there was some delay with resolving the team leaders' requests since not all the time during the moderation the assigned 3 IB members were connected to the Zoom meeting and not always responded quickly to the e-mailed requests to join this meeting. Therefore, to facilitate the whole procedure in case of a future remote IPhO, we would advise the 3 IB members to agree between themselves that at least one of them should always be present in the Zoom meeting. Also, for faster response the usage of Instant Messengers by the future organisers and the IB representatives could be beneficial.

Organising of online events to engage students, team leaders, and observers

Many different freetime activities were offered during the IPhO 2021. The activities were created for the IPhO 2021 participants to communicate with each other, learn something new, get to know Lithuania, and have some fun. The following activities were organised. The activities usually were organised twice to account for the time differences among the participating countries.
Social networking activities for the students to get acquainted with each other. Quite a large number of students participated in this event and were divided into groups to communicate with other students from different countries and volunteers.
Virtual tourism activities were organised for students, team leaders and observers. It was an online tour about Lithuania to present the IPhO 2021 host country's history, culture and nature to the participants.
The lecture of the Nobel prize winner Professor Didier Queloz "The Exoplanet Revolution" was organised exclusively for the students. After the lecture, the students had a chance to ask questions. The discussion was very active and there was not enough time to answer all the questions that were asked. The recording of the lecture was also made available for the IPhO 2021 participants to view later if they were not able to participate during the live broadcast.
Mindfight games (trivia games) were organised for the students, the team leaders and the observers. Students in local or international groups tried to answer various questions. The best teams received small gifts. Only 2 of the team leaders and observers joined the Mindfight game session dedicated to them, so it was rescheduled and later made into a game for all IPhO 2021 participants.
Lithuanian Dance and Music workshop was organised for the students. Vilnius University folk ensemble "Ratilio" presented the Lithuanian folk dances and games. Some students danced in their home countries, while the others just watched the

performance.

Lithuanian food workshop was organised for the students. Šaltibarščiai (cold beetroot soup) was made during a Zoom broadcast and the students learned how to do it themselves. Even though no one except the volunteers tried to make Šaltibarščiai during the activity, a lively discussion about the Lithuanian cuisine followed the presentation.

Science day with four different lectures of Lithuanian scientists was organised for the students. Topics about light, lasers, astrophysics and particle physics were covered. Students had a possibility to participate in all lectures or only in the selected ones.

Lithuanian movie "Emilia. Breaking free" was available for all IPhO 2021 participants to watch at any convenient time from the 21th till 24th of July. Discussion about the movie with a Lithuanian historian Dr. Simonas Jazavita was organised for the students only to keep it more familiar. Even though not many students participated, discussion was quite active and students asked a lot of questions about the Lithuanian history.

For **teamwork presentations** students were randomly divided into groups with at least one student volunteer. Together as a team, they were presented with a task to prepare a performance of a given physics phenomena without using any words. This challenge required a lot of creativity and all the teams succeeded to create a performance. Other teams were able to guess what the performance was about for extra points. The points were also given by the jury of organisers for creativity and co-operation of the whole team. The best three performances won small prizes.

Chess tournaments were organised for the students. The lichess.com platform was used for the tournaments. The tournaments lasted for 1,5 hours and could be viewed by anyone. In the main chess tournament around 20 students participated and the winners received small prizes.

During the IPhO 2021 students and their team leaders were unfortunately not able to experience Lithuania physically by themselves, so **Geoguesser** game about Lithuania was organized for them. With the help of the street view technology participants were able to guess which big city or tourist attraction of Lithuania they were seeing.

Use of a social networking platform

In order to facilitate the IPhO 2021 organisation process as well as in order to create a space for some off-topic message exchange, a social networking space was created. The main objectives of the social networking medium were:

- Creating a safe space for the students and all other participants to connect;
- Facilitating the introduction of students to their assigned volunteers;
- Keeping the off-topic discussions in a separate channel from the main communications;
- Informing all the participants about any last minute changes quickly without having to call people.

We decided that a messaging platform would be sufficient to achieve these objectives. In the survey about the communication tools we asked what messaging services were not blocked in the participating countries. Based on the survey results, WhatsApp turned out to be the least blocked service among the participating countries. For this reason we chose WhatsApp as the default messaging service.
However, we experienced a major challenge because we did not foresee that the WhatsApp group can have a maximum of 256 participants. This was too small for all (or at least the majority) of the participants.
We then switched to a Telegram service which offered much bigger group sizes. Telegram was the second most available service among the participating countries. However, as anticipated, migrating everyone to a new service seemed problematic and in the end we ended up using both: WhatsApp and Telegram for important announcements and reminders.

Online Newsletters

Online IPhO 2021 newspaper “Share the light” was prepared and released on each day of the Olympiad. There were 9 issues in total. Each of them had a theme. The articles as well as some information about the Olympiad, IPhO participants’ interviews and quizzes were related to the theme of the day. The following themes were covered:

- ISSUE 1: Facts about IPhO and Lithuania,
- ISSUE 2: Lithuania’s place in the (scientific) world,
- ISSUE 3: Scientists,
- ISSUE 4: Culture,
- ISSUE 5: Nature & climate,
- ISSUE 6: Lithuanian women in science,
- ISSUE 7: Food,
- ISSUE 8: Studies and innovation in Lithuania,
- ISSUE 9: IPhO 2021 results & photos.

Some interviews of the IPhO 2021 participants were collected before the Olympiad while others were made during the Olympiad. Organisers also got some photos from the teams, which were added to the Newsletters. The last issue covered the results of the IPhO 2021 and some moments of it. Some content was created before the start of the Olympiad while the rest was written during the event.
Newspapers were published on the www.ipho2021.lt page as well as sent in a daily email to all participants.
The daily emails were sent to all participants from July 16 to July 24. The students did not get daily emails during the days they were supposed to have their communication devices taken away from them.

Each daily email was sent in 3 versions:

- For students;
- For team leaders;
- For invigilators.

Each email had the following structure:

- One or two most memorable things that happened that day;
- Link to the content if it's available for later use (e.g. lecture recordings);
- The link to the newest IPhO 2021 Newspaper;
- The activities for tomorrow with the access links and times;
- The reminders about the multi-day activities such as the movie, game access, etc.

Payments and financials

The total IPhO 2021 cost was about 800 000 EUR (or roughly 888 000 USD). This number does not include the accurate monetary value of the services provided by the salaried workers of the organizing institutions nor the services provided as volunteering (see Figure 1). Therefore, the above number is somewhat inaccurate to reflect the amount of work that was put in. In addition, this number includes inventories which were purchased for the physical version of the IPhO 2021 but were unused due to the changed circumstances.

Figure 1.

The major part of the costs were incurred for the direct expenses (730 000 EUR). The structure of the direct expenses is explained in the Figure 2. The earliest expenses include the observer participation in the IPhO 2016. The preparation groundwork for IPhO 2021 began in 2018. Then the first budget was calculated and the first public procurement procedures were initiated.

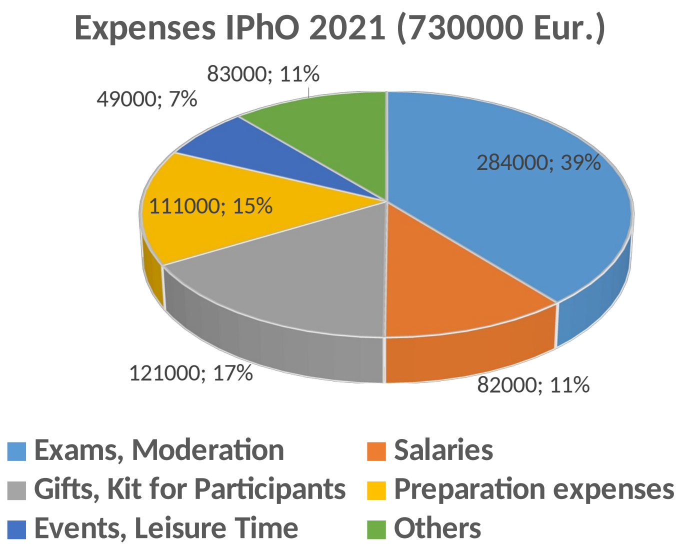


Figure 2.

About 40% of the IPhO 2021 expenses were attributed to the theoretical and experimental exams (about 284 000 EUR). This amount includes the creation of the problems, the manufacturing of the experimental exam equipment, exam moderation and marking, salaries for the academic workers associated with the moderation and marking. Vilnius University's financial direct contribution to the event was 186 500 EUR.

We approximately evaluate the Vilnius University indirect costs for the event organization (rooms, IT department working hours, dean’s office working hours and other employees) – 30 000 EUR. Vilnius University collected 24 391 EUR from various sponsors which were dedicated for the Academic committee’s needs to reduce Vilnius university costs. See the list below for the details of the Academic Committee expenses.

Category	Amount, EUR
Authors’ salaries	40 000
Remuneration for Markers and Moderators	85 000
Experiment preparation (100 000 EUR received from the government without VAT)	135 000
Printing and scanning (without paper and	5 000

office supplies)	
Internet upgrade and WIFI	1 500
OlyExams programs	9 000
Organisational and administrative	6 000
Other expenses	5 000
Total:	286 500
Totally without government input:	186 500

The second largest expense for the IPhO 2021 was the gifts for the participants (backpacks, T-shirts, water bottles, etc.) and gifts for the winners. This part does not include the experimental exams (which were sent in the same shipment). The gifts cost about 121 000 EUR or 17% of the direct expenses.
Other large parts of the direct expenses included the preparation expenses (15%), the salaries (11%) and other expenses (11%). This expense distribution clearly shows that the event was online. Had it been in the physical format the distribution would have greatly shifted and the overall cost would have reached probably about 1 500 000 EUR.

Let's now have a look at where the funding for the IPhO 2021 came from (Figure 3). The largest contribution was made by the Ministry of Education, Science and Sport of the Republic of Lithuania – 340 000 EUR or about 59% of all organiser contributions. Vilnius University also made a large monetary contribution – about 186 000 or about 32% of the organiser contributions. The Lithuanian Centre of Non-formal Youth Education contributed about 50 000 EUR.
The collected participation fees covered about 25% of the income.
Due to the cultural norms, the event sponsorships in Lithuania are rarely in a monetary form. More often they take a form of an in-kind support or special discounts for the organizers. As businesses pay relatively high taxes they expect the government to cover the expenses of such public events. Therefore the monetary sponsorships were negligible in the context of IPhO 2021.
All the financial operations were performed by the Lithuanian Centre of Non-formal Youth Education with the exception of the academic activities which were taken care of by the Vilnius University.

Figure 3.

Volunteer taskforce

A team of 42 volunteers were helping during the days of the IPhO 2021. Almost all of them were gathered in the Physics Faculty of the Vilnius University, where all of the organisers were located. Volunteers were divided into five groups:

- 17 students' volunteers who were responsible for the communication with the students from 4-5 countries. The same volunteers also helped during the leisure activities as well as to supervise the exams;
- 5 team leaders' volunteers who were responsible for the International Board meetings and also helped during the printing and moderation;
- 7 Academic Committee volunteers who mostly helped with the technical support regarding the OlyExams, experimental setup troubleshooting as well as printing and distributing students' exams solutions to markers;
- 7 Organising Committee volunteers who helped to ensure a smooth conduction of all leisure activities. They also helped to supervise exams;
- 6 media volunteers who helped with IPhO 2021 newsletter and social media posts. This group of volunteers started working before the Olympiad.

Volunteers' registration and selection process started in March, 2021. Many volunteers were recruited from the lists of volunteers who signed up for the attempted IPhO 2020. Some volunteers helped to prepare the experiments for shipping in April and May. An official volunteer training for the Olympiad was held online one week before the Olympiad.

Online Opening and closing ceremonies

IPhO 2021 Opening Ceremony

Due to high uncertainty of the IPhO 2021 format the Opening Ceremony had to be replanned and reinvented several times. The final decision was reached to pre-record the Opening Ceremony a few days in advance to avoid the last minute works piling up and to avoid any technical difficulties.

The IPhO 2021 was opened by the President of the Republic of Lithuania Gitanas Nausėda and the Minister at the Ministry of Education, Science and Sport of the Republic of Lithuania Jurgita Šiugždinienė.

After the formal greetings a short video clip made by Jokūbas Laukaitis was played to give an introduction about Lithuania – the host country of the IPhO 2021.

During the Opening Ceremony a huge focus was given to the history of IPhO, the Organisers of the IPhO 2021, the preparation works were shown. One of the official organisers of the IPhO 2021 is Vilnius University, therefore the virtual floor was given to the Rector of Vilnius University prof. Rimvydas Petrauskas.

Later, all participating delegations were introduced (students, team leaders, observers and invigilators).

All the delegations were greeted by the IPhO President prof. Rajdeep Singh Rawat. He also officially opened the International Physics Olympiad 2021.

The Opening Ceremony also included some of the top Lithuanian performers. The beginning of the ceremony was marked by Vidas Bareikis. The ceremony finished with the group Subtilu Z video clip "Watermelon man".

IPhO 2021 Closing Ceremony

The IPhO 2021 Closing Ceremony faced the same challenges as the Opening Ceremony. The changing IPhO 2021 format influenced the content and the delivery in major ways. Due to the technical challenges and limited resources, the Closing Ceremony was recorded the day before it went live.

The Closing Ceremony included the remarks from the Chairman of the Academic Committee Vilnius University professor Edmundas Kuokštis who also mentioned some insights about creating the exam problems. The winners were congratulated by the Deputy Minister of the Ministry of Education, Science and Sport of the Republic of Lithuania doc. dr. Ramūnas Skaudžius and the IPhO President prof. Rajdeep Singh Rawat.

All the winners of the medals as well as the receivers of the honourable mentions and special prizes were presented. A video clip with the memories from the past days was shown.

At the end of the Closing Ceremony the Minister of Education of Belarus, Igor

Karpenko, invited all the participants and the Physics community to join the 52nd International Physics Olympiad (IPhO 2022) in Minsk, Belarus.
Just like the Opening Ceremony, the Closing Ceremony presented the creativity of some of the best artists of Lithuania. The celebration included Martynas Levickis, one of the most talented accordion players in Lithuania who is widely known around the world. The viewers saw the video clip of Justinas Jarutis and Jessica Shy as well as the performance of one of the Eurovision 2021 favourites – Lithuanian band the Roop. The end of the ceremony was marked by the Lithuanian folk-rock band Žalvarinis.

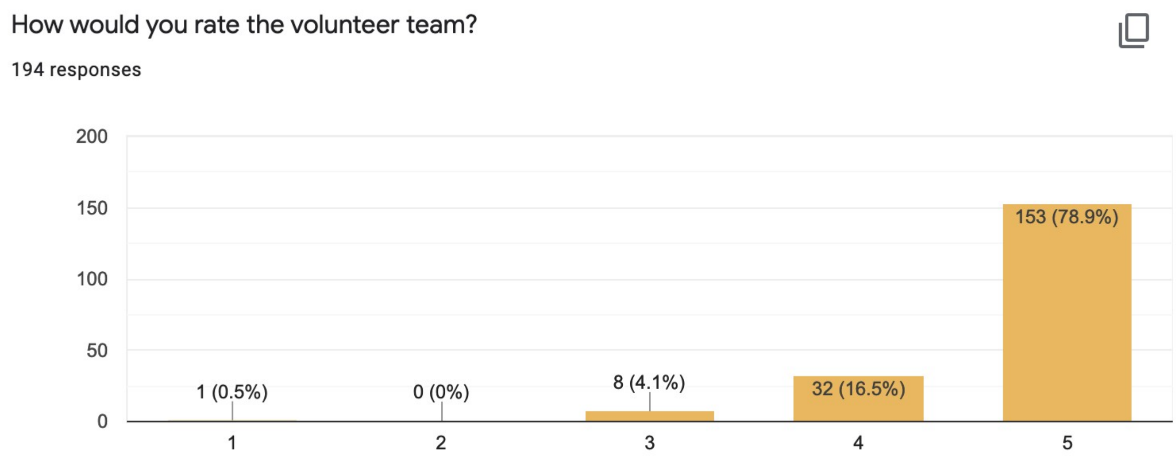
The Opening and Closing Ceremonies were broadcast over YouTube platform. As of this day the Opening Ceremony has been viewed 11 000 times, the Closing Ceremony has been viewed 13 000 times.

Feedback

After the IPhO 2021 finished we launched a feedback survey. The survey had the following questions:

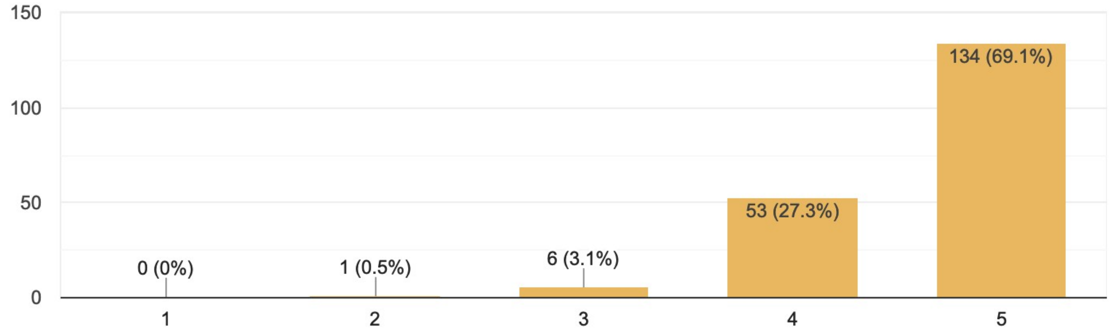
- What is your country or region?
- What is your role in the IPhO 2021?
    - Team leader
    - Observer
    - Student
    - Invigilator
- Statement: The next questions are all on a scale where 5 is the best and 1 is the worst.
- How would you rate the whole IPhO 2021 experience?
- How would you rate the volunteer team?
- How would you rate organizing and academic teams?
- What did you like the best about the IPhO 2021?
    - Freetime activities
    - Exams (students and invigilators)/Formal meetings (team leaders and observers)
    - Opening or closing ceremonies
    - WhatsApp, Telegram, Informal freetime space (for students)
- What part could be improved about the IPhO 2021?
    - Freetime activities
    - Exams (students and invigilators)/Formal meetings (team leaders and observers)
    - Opening or closing ceremonies
    - WhatsApp, Telegram, Informal freetime space (for students)
- Other comments

We received 194 responses from 73 countries. About 50% of the responders were students, about 25% were the invigilators, about 20% were the team leaders, while the rest were classified as 'others'.
The results are the following:



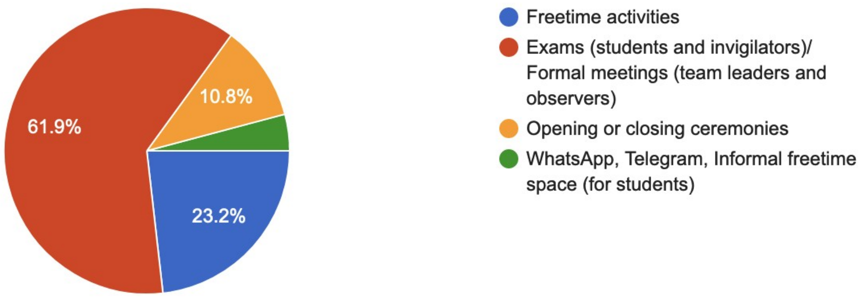
How would you rate organizing and academic teams?

194 responses



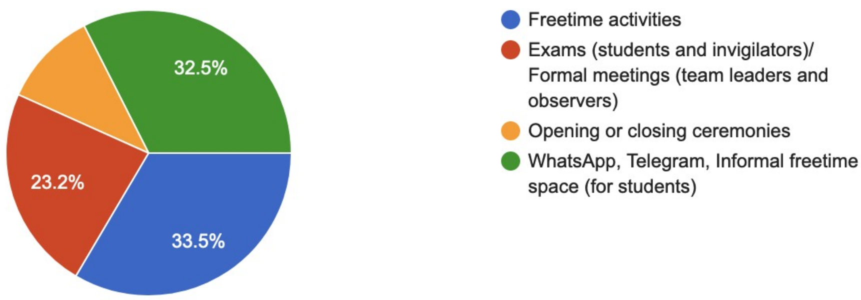
What did you like the best about the IPhO 2021?

194 responses



What part could be improved about the IPhO 2021?

194 responses



In the other comments we saw a positive trend of people being happy about the event. There were several explanations that some participants were disappointed and gave bad scores because the IPhO 2021 was organized online instead of physically. Some people had troubles with the tablets during the experiments or while preparing them for the experiments. Many participants enjoyed the trivia game–Mindfight. Several people expressed the difficulties to manage the time zone difference for the event.

Conclusions

All in all, the 51st International Physics Olympiad faced numerous challenges and uncertainties due to the threats from the COVID-19 pandemic. A one year gap was necessary to evaluate all the risks and changes caused by this global phenomenon. However, we can proudly say that judging by the feedback the IPhO 2021 was a smashing success. We judge that it was organized in the best way it was possible in the given situation.

It was the first ever International Physics Olympiad hosted virtually. Numerous new procedures were introduced which were later adopted by the other science olympiads and competitions.

The fraud prevention measures were rather strict but as organizers we felt they were needed to ensure the fairness and transparency of such a highly respected event.

Of course, not all the technologies worked and sometimes it seemed like there was no best way to address the challenges. Quite notably, the Zoom meeting stopped functioning properly during an exam, the WhatsApp group turned out to have a limit on group members, and some delegations had huge problems with internet speed and connectivity as well as with electricity supply. Despite all this, the Olympiad happened and judging by the feedback it was viewed very positively.

As organizers we can give the following suggestions for the future olympiads:

- Create and enforce deadlines more strictly;
- Give a lot of consideration for shipping anything ahead of the event;
- Prepare yourself that some things will not work;
- Don’t wait too long to make a decision about the format of the event–you will save a lot of planning and preparations time;
- Always try to make the decisions based on some hard data. Surveys are one way of doing it.

In the end, we wish all the future organisers the best courage and inspiration to continue the IPhO traditions, bring people closer, have fun and exciting examinations as well as free time, and above everything – unite the physicists of the world and share the light!

Annex

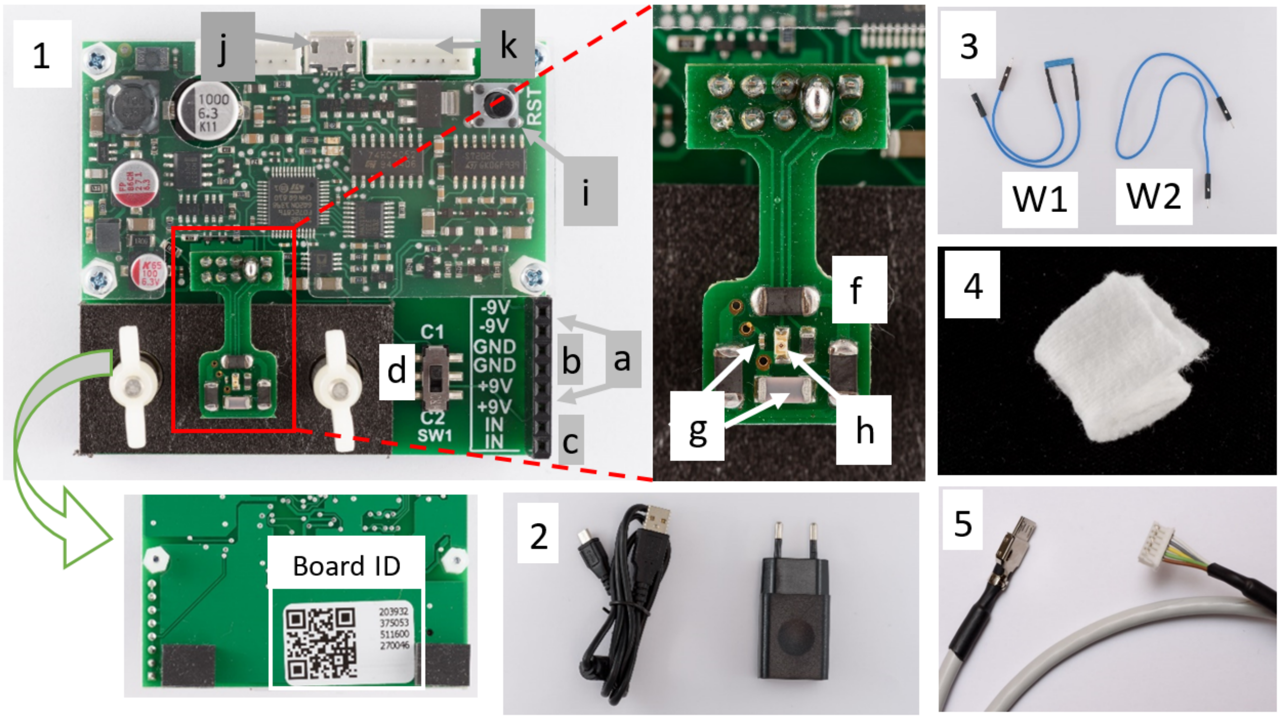
Experiment Exam Guide
Experiment Exam Problem 1
Experiment Answer Sheet 1
Experiment Exam Problem 1 Solution
Experiment Exam Problem 2
Experiment Answer Sheet 2
Experiment Exam Problem 2 Solution
Theory Exam Problem 1
Theory Answer Sheet 1
Theory Exam Problem 1 Solution
Theory Exam Problem 2
Theory Answer Sheet 2
Theory Exam Problem 2 Solution
Theory Exam Problem 3
Theory Answer Sheet 3
Theory Exam Problem 3 Solution

Experimental Examination - Overall Guide

The experimental examination lasts 5 hours and consists of 2 separate experiments worth 10 points each. Equipment is partially shared between two experiments, so read these instructions carefully before starting your work.

Equipment list:

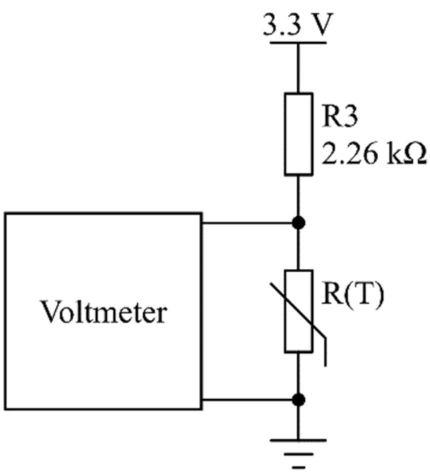
1. Measurement and sample board containing:
   (a) +9 V and -9 V constant voltage source (two equivalent terminals available for each),
   (b) Two equivalent ground terminals,
   (c) Two equivalent capacitor terminals,
   (d) Capacitor selection switch (can be set to C1 or C2),
   (e) Voltmeter with low input current (in the board),
   (f) Thermostat with heater and temperature sensor (in the board),
   (g) Sample capacitors C1 and C2,
   (h) LED connected to a constant current source and voltmeter,
   (i) RESET button,
   (j) USB power port,
   (k) 6-PIN data port for connecting to the tablet.
2. Power source for the board with USB Micro-B plug.
3. Jumper wires – W1 (with 100 MΩ resistor R1 inside) and W2 (0 Ω).
4. Heat insulating material for the thermostat.
5. Connector cable between the board and tablet, with USB Micro-B plug on the tablet side.
6. Touchscreen tablet running IPhO 2021 Experiments app (app user manual provided below).
7. Thermometer (available in examination hall).



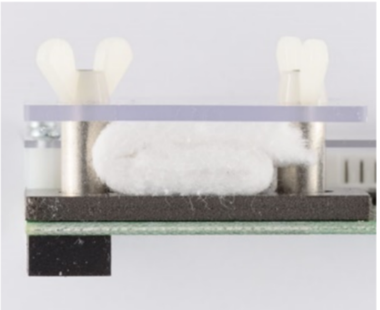
Thermostat's temperature is measured using NTC (Negative Temperature Coefficient) thermistor, its resistance depends on absolute temperature T (in Kelvin) as follows:

$$R(T) = R_0 e^{B/T}, \tag{1}$$

$B = 3500\ \mathrm{K}$, R_0 – constant, has to be calculated from the known environment temperature before turning on the heating. The value of this constant is necessary for both experiments. The temperature of the thermostat can be controlled by changing the heating current (via the app). After changing the heating current, it is necessary to wait to let system reach a stable temperature. On the other hand, the thermal equilibrium between the components (capacitors, NTC and LED) is assumed to happen "instantly", and no significant delay is observed.



To ensure more stable thermal conditions, a layer of insulating material has been placed over thermostat and pressed onto it using a small plastic plate held by two screws.



Caution:
Avoid damaging the board and sockets on it, make sure you're plugging everything correctly without excessive force.
Liquids do not mix well with electronics, so be careful while handling liquids (like drinking water) near the experimental setup. Don't accidentally spit on it.

IPhO 2021 Experiments app user manual

IPhO 2021 Experiments software can be launched from tablet's home screen (or from an app drawer, accessible by swiping screen from bottom to top) by tapping on IPhO icon.

In order to get values measured on the board to the tablet:

1. power the board using USB charger;
2. connect the board and the tablet using the connector cable (*6-pin on the board side and Micro-USB on the tablet side);*
3. confirm the USB access and reset the board in 10 seconds when the app asks you to do so.

Caution: if at some moment

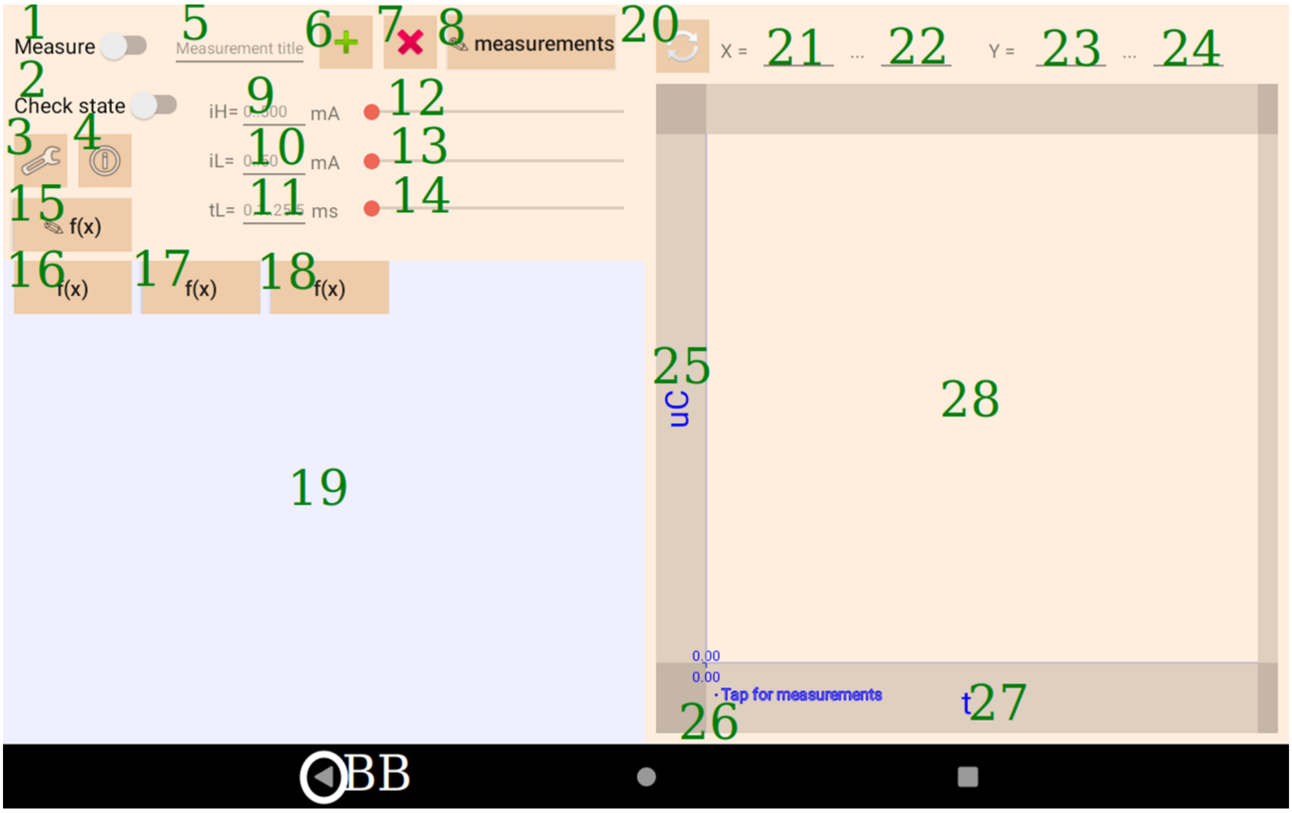
- **the board stops responding and returns no measurements (in either "Check state" or Measurement mode),**
- **heating / LED current has no change (thermistor voltage does not change and LED does not glow even at maximum LED current),**

press RESET button on the board and do the step 3 "confirm the USB access...".

If it does not help:

- **exit the app by tapping the Back button twice,**
- **unplug the board,**
- **open the app again,**
- **reconnect the board again and do the step 3 described above.**

Controls and fields are (the numbers will be used as references later):



The main window of the app.

- **1 –** Tapping this toggle starts a measurement session. Tapping it again stops it.
- **2 –** When this toggle is selected, the screen shows the live values of the measurements.
- **3 –** Opens settings.
- **4 –** Pops up short summary of settings.
- **5 –** Measurement title to be saved or deleted.
- **6 –** Saves a newly measured or selected measurement under a new name.
- **7 –** Deletes selected measurement.
- **8 –** Selects a previously saved measurement.
- **9, 10, 11 –** Text fields to manually enter heating current (**9**), LED current (**10**), LED current pulse duration (**11**) values. Empty values mean 0. **tL** (LED current pulse duration) **= 0** means constant direct current).
- **12, 13, 14 –** Seekbars to change the corresponding values (*LED current changes exponentially!*).
- **15 –** Opens functions editor.
- **16, 17, 18 –** Selects variables or functions for measurement table columns.
- **19 –** Measurement table area.

- **20 –** Manually replots measurements in a chart.
- **21, 22 –** *X* axis min and max limits (can be entered manually and replot button pressed).
- **23, 24 –** *Y* axis min and max limits.
- **25, 27 –** selects *Y* and *X* axes of the chart.
- **26 –** selects measurements to be plotted on the chart.
- **28 –** chart area.
- **BB –** the Android OS Back button (tap twice to close the app).

Setting up a sweep I-V curve measurement

Additional LED controls are available for the LAB 2 by tapping the settings button (**3**) of the main window. **In the window that opens select:**

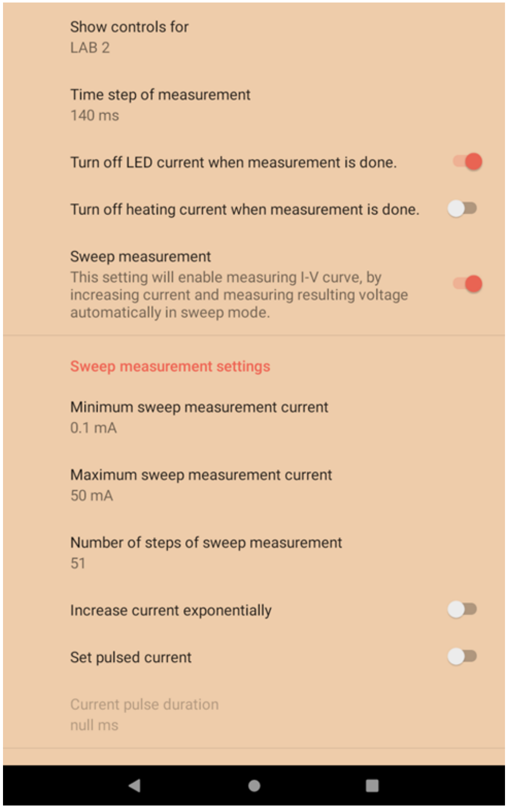
- ”LAB 2” or “ANY LAB” in the ”**Show controls for**” section.
- activate the ”**Sweep measurement**” option.

Other settings are:

- ”**Minimum...**” and ”**Maximum sweep measurement current**” set the starting value and the last value of LED current during sweep measurement respectively.
- ”**Number of steps of sweep measurement**” means how much measurement steps will be made.
- choose ”**Increase current according to geometric progression**” if you want current to increase exponentially.
- choose ”**Set pulsed current**” and set ”**Current pulse width**” if you want each value to be measured using limited time pulse of LED current.

*E.g., if the number of steps is 51, “**Increase current according to geometric progression**” is off, LED current changes from 0 mA to 50 mA respectively, the LED current during measurement will be 0 mA, 1 mA, ... 49 mA and 50 mA.*

Now you can start measuring I-V curve after returning back to the main window by pressing the Back button.



Editing functions

Tapping the (**15**) button of the main window opens functions editing window.

The functions created can accept some of the variables (and their derivatives) directly measured on the board.

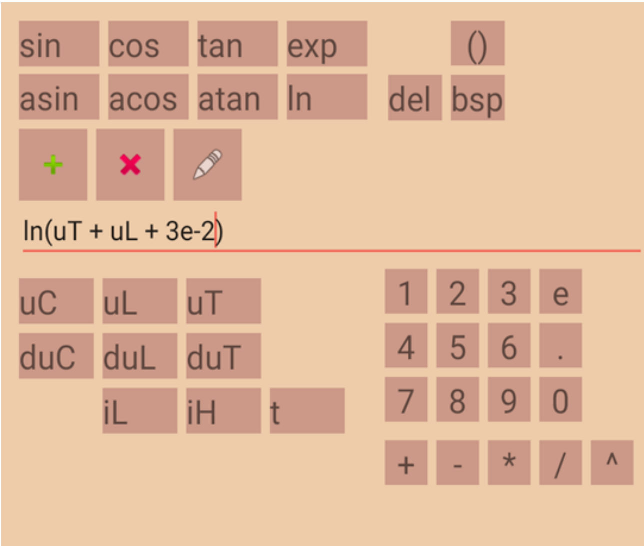
They are:

- **voltages (in V):**
  - **uC** – at the selected capacitor (C1 or C2);
  - **uT** – at the thermistor;
  - **uL** – at the LED;
- their derivatives with respect to time ($\mathrm{d}y/\mathrm{d}t$) (in V/s):
  - **duC**
  - **duT**
  - **duL**
- the currents (in mA):
  - **iL** – at LED (in mA);
  - **iH** – heating current (in mA);
- time **t** (in s).

It is possible to enter a custom function using these variables and mathematical functions (using helper buttons or a standard Android keyboard) of your choosing and save it by pressing a **green +** button afterwards. The saved functions can be used as the graph axes or as the measurement table columns. The pencil button selects existing functions. The selected functions can be deleted by pressing **red x** button.

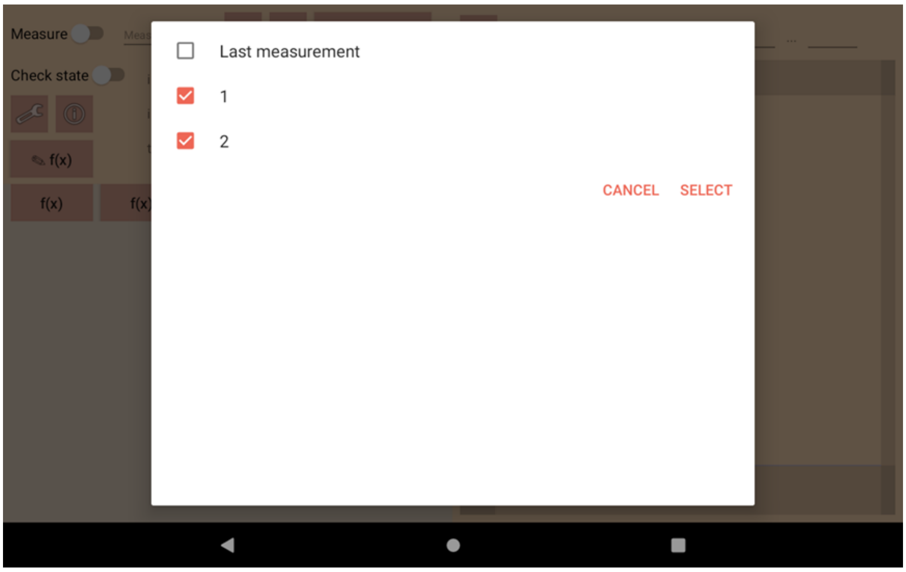
Both casual decimal format (e.g. **25.02**) and scientific format (e.g. **2.502e+1**) are acceptable for numbers.

- ***** is multiplication operator,
- / is division operator,
- ˆ is power operator.

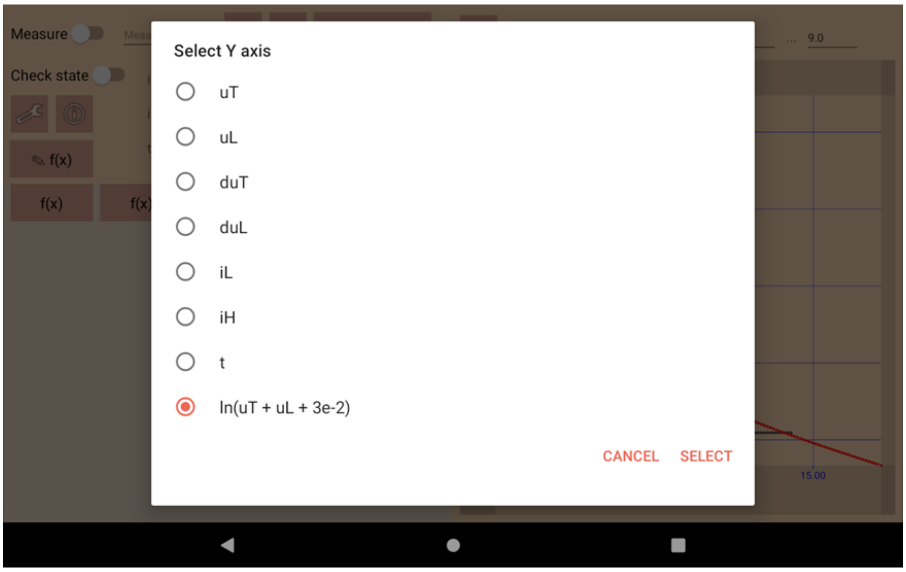


Viewing measurements

The finished measurement can be saved by entering its name into the (**5**) field in the main window and pressing a **green +** button (**6**) nearby. The raw measurement data is saved, which can be displayed on any other axes later. The saved measurements can be displayed on the chart by tapping area near the corner of the chart (**26**).



You can pan/zoom the chart, and if you tap it at the exact point, either the closest point of the measurement (if no close measurement points to the tapped exist) or the point itself will be marked and its coordinates displayed.



Axis can be chosen by tapping existing axis labels (chart areas **25** and **27**).

Non-ideal capacitors (10 points)

This experiment is designed to investigate the properties of capacitors.

Capacitor's capacitance (which always means differential capacitance in this text) can be found based on its charging graph of its voltage $U(t)$ via the resistor R_1. Depending on the circuit, it is necessary to find the relation of capacitor's charging current vs voltage $I(U)$ and use it to determine capacitance:

$$C(U) = \frac{dq}{dU} = \frac{I dt}{dU} = \frac{I(U)}{dU/dt}. \tag{1}$$

The electric circuit implemented in this experiment is shown in Fig. 1.1. Switch S1 on the board can be used to switch between capacitors C1 and C2. The middle position of the switch does not play any role in this experiment and should never be used.

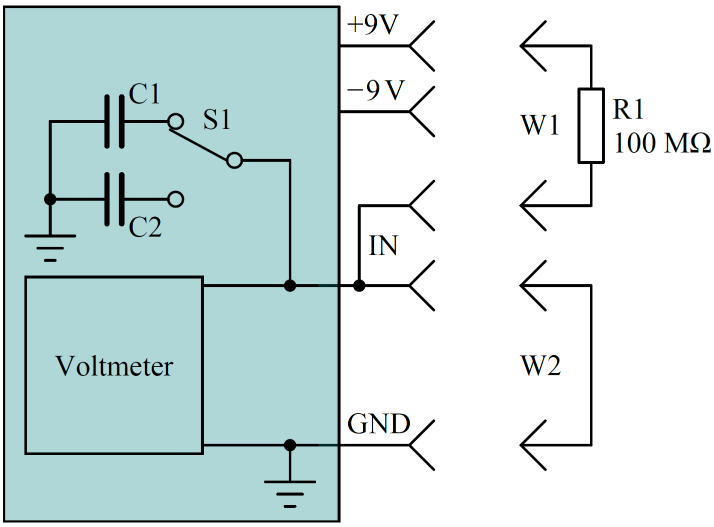


Figure 1.1. Electric circuit for the experiment.

Caution: one of the sample capacitors contains a dielectric with dielectric permittivity that depends on the capacitor voltage change rate. To keep this rate as stable as possible, when measuring at the positive voltages, the capacitor should be charged from 9 V down to –9 V, while measurements at the negative voltages should be done when capacitor is charged from –9 V towards 9 V. The measured capacitance can be influenced by the previous state of the capacitor, thus capacitor should be kept at the starting voltage for at least 10 s before the measurement.

Part A. Capacitors at room temperature (4.0 points)

Measure and graph the capacitance of the capacitors C1 and C2 versus the voltage at room temperature (draw all graphs together on the same axes).

A.1	Measure and graph $C_1(U)$ and $C_2(U)$ in range from –7 V to 7 V. In the answer sheet write C_1 and C_2 values at 0 V, 3 V, and 6 V. Write down the formula used for calculating capacitance from raw measurements. Also write Board ID and room temperature.	2.3pt

A.2	Find the voltage $U_{\text{max change}}$, at which the capacitance of capacitor exhibits the fastest relative change versus the voltage $\left(\frac{\mathrm{d}C(U)}{C(U)\mathrm{d}U}\right)$. In the answer sheet write, which capacitor (C1 or C2) exhibits the fastest change and the voltage, at which it is observed.	0.5pt

A.3	What are the charges q_1 and q_2 of capacitors C1 and C2 at 6 V?	1.2pt

Part B. Calibrating NTC thermistor (1.0 point)

Measure the NTC (Negative Temperature Coefficient) thermistor voltage at a known room temperature (from examination hall thermometer). The formula (1) for its resistance vs temperature and it's circuit is show in "Experimental Examination - Overall Guide G1".

B.1	Find the NTC thermistor constant R_0.	1.0pt

Part C. Capacitors at different temperatures (3.0 points)

C.1	Measure and graph $C_1(U)$ and $C_2(U)$ in range from –7 V to 7 V at temperatures of 40 °C, 65 °C and 85 °C.	1.3pt

C.2	Graph $C_1(T)$ and $C_2(T)$ at 0 V and 6 V versus temperature from room temperature up to 85 °C.	0.5pt

C.3	In the answer sheet write the ratio $C(85\,^{\circ}\mathrm{C})/C(40\,^{\circ}\mathrm{C})$ for both capacitors C1 and C2 at 0 V and 6 V.	1.2pt

Part D. Sources of measurement errors (2.0 points)

The previous tasks in this experiment were done in conditions of long initial charge. When looking at shorter recharging times (0.1 - 10 s) there can be multiple sources of errors:

1. Leakage current.

2. Polarization properties of the capacitor's dielectric media that can be expressed as the dielectric permittivity that depends on process time scale.

Caution: heat-insulating material may absorb air moisture and become conductive. Remove it when doing leakage measurements.

Determine the main source of error for measuring C1 and C2, since capacitor leakage and voltmeter input currents depend on the voltage, estimate these errors at voltage close to 9 V. Decide, which auxiliary measurements and under what conditions should be taken in order to answer these questions. In your answers to the following D.1 and D.2 questions, you might indicate the conditions of your measurements, which quantities you measure and what conclusions you make based on your measurements, as exemplified in the tables below.

Note: these are just the examples how to describe schematically your measurements; you need determine the relevant conditions of your measurements by yourself.

Examples of how answers to questions D.1 and D.2 should be written:

Example 1.
Showing that voltage change rate of C1 connected to the measuring circuit is faster at 9 V than at 0 V.
Possible S1 positions: C1, C2
Possible IN connection: +9V, -9V, GND, Free
Initial settings:

S1 position	IN connection
C1	9V

Process:

Step number	S1 position	IN connection	Duration, s	Measured variable
1	C1	Free		$\|duC(t)\|/dt$
2	C1	GND		
3	C1	Free		$\|duC(t)\|/dt$

Verification: $|duC(t)|/dt|_1 > |duC(t)|/dt|_3$

Example 2.
Showing that voltage change rate of C1 at 9 V is larger than the average voltage change rate starting at 0 V over 1000 seconds.
Possible S1 positions: C1, C2
Possible IN connection: +9V, -9V, GND, Free
Initial settings:

S1 position	IN connection
C1	9V

Process:

Step number	S1 position	IN connection	Duration, s	Measured variable
1	C1	Free		$\|duC(t)\|/dt$
2	C1	GND		
3	C1	Free		uC
4	C1	Free	1000	
5	C1	Free		uC

Verification: $|duC(t)|/dt|_1 > (uC|_3 - uC|_5)/1000$

D.1	What is the main source of error for measuring $C_1(9\,\mathrm{V})$? Write the measurement steps in the tables.	1.0pt

D.2	What is the main source of error for measuring $C_2(9\,\mathrm{V})$? Write the measurement steps in the tables.	1.0pt

Non-ideal capacitors (10 points)

Part A. Capacitors at room temperature (4.0 points)

A.1 (2.3 pt)
Graph $C_1(U)$ and $C_2(U)$ on the millimeter paper:

Fill the table with the corresponding values:

U	C_1	C_2
$0V$		
$3V$		
$6V$		

$C(U) =$

Experiment conditions:

Board ID =

$T_{\text{room}} =$

A.2 (0.5 pt)

$U_{\text{max change}}$ =

At capacitor (check): ☐ C1 ☐ C2

A.3 (1.2 pt)

q_1 =

q_2 =

Part B. Calibrating NTC thermistor (1.0 point)

B.1 (1.0 pt)
Formula:

R_0 =

Value:
R_0 = Ω

Part C. Capacitors at different temperatures (3.0 points)

C.1 (1.3 pt)

Graph $C_1(U)$ and $C_2(U)$ on the millimeter paper:

C.2 (0.5 pt)
Graph $C_1(T)$ and $C_2(T)$ on the millimeter paper:

C.3 (1.2 pt)
$C_1(85°\mathrm{C})/C_1(40°\mathrm{C})|_{0\,\mathrm{V}}$ =

$C_1(85°\mathrm{C})/C_1(40°\mathrm{C})|_{6\,\mathrm{V}}$ =

$C_2(85°\mathrm{C})/C_2(40°\mathrm{C})|_{0\,\mathrm{V}}$ =

$C_2(85°\mathrm{C})/C_2(40°\mathrm{C})|_{6\,\mathrm{V}}$ =

Part D. Sources of measurement errors (2.0 points)

D.1 (1.0 pt)
Possible S1 positions: C1, C2
Possible IN connection: +9V, -9V, GND, Free
Initial settings:

S1 position	IN connection

Process:

Step number	S1 position	IN connection	Duration, s	Measured variable

Verification:

Main source of error (check):
☐ Leakage current
☐ Polarization properties of the capacitor's dielectric media

D.2 (1.0 pt)
Possible S1 positions: C1, C2
Possible IN connection: +9V, -9V, GND, Free
Initial settings:

S1 position	IN connection

Process:

Step number	S1 position	IN connection	Duration, s	Measured variable

Verification:

Main source of error (check):
☐ Leakage current
☐ Polarization properties of the capacitor’s dielectric media

Non-ideal capacitors (10 points)

Capacitance measurement method:

First, measure the highest voltage the capacitor can reach by connecting it to the voltage source via jumper wire W2. Before each measurement, connect capacitor to starting voltage source with jumper wire W2 and to a final voltage source (U_f) with jumper wire W1 via the resistor R1. Capacitor C2 should be prepared that way for at least 10 s, while C1 measurement can be started immediately by disconnecting jumper wire W2 from the starting voltage source. To determine a precise value of the final voltage U_f, it should be measured after capacitor has been connected to final source via R1 for a long time (at least 3 minutes). Then, the capacitance can be calculated from:

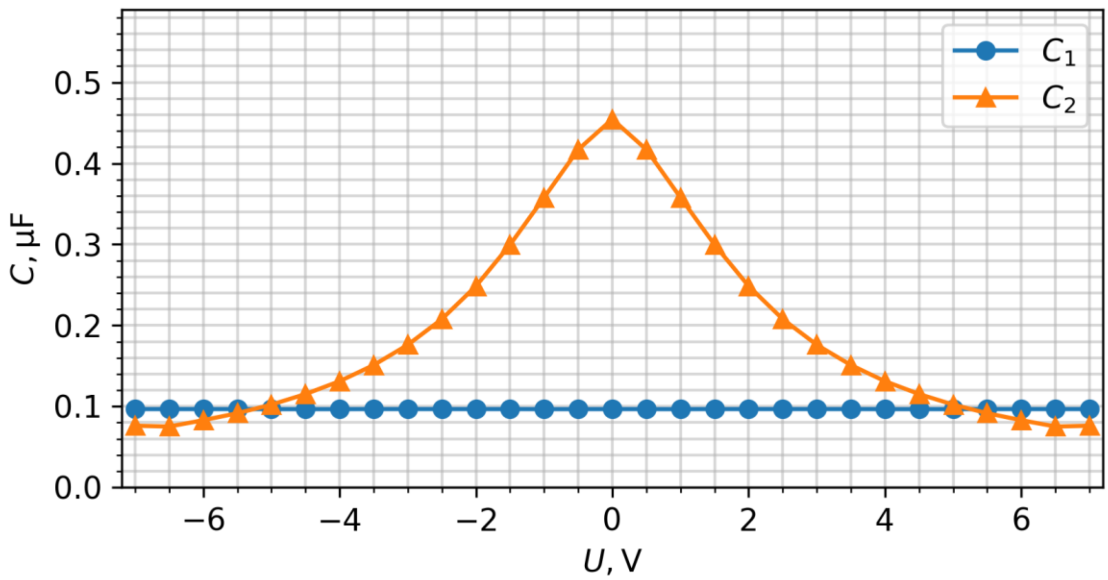
$$C(U) = \frac{U_f - U(t)}{R1} \Big/ \frac{\mathrm{d}U}{\mathrm{d}t}$$

When measuring C2, to ensure minimal change in charging current, capacitance should only be calculated in conditions where U_f and $U(t)$ have different polarities. This way, capacitance dependence on voltage should be symmetrical around 0 V.

Part A: Capacitors at room temperature (4 points)

A.1 (2.3 pt)

Graph $C_1(U)$ should be constant, $C_2(U)$ must be highest at 0 V.
Example results measured at room temperature of 29 °C.



	C_1	C_2
0 V	0.100 μF	0.473 μF
3 V	0.100 μF	0.183 μF
6 V	0.100 μF	0.086 μF

$$C(U) = \frac{U_f - U(t)}{R1} \bigg/ \frac{\mathrm{d}U}{\mathrm{d}t}.$$

A.2 (0.5 pt)

$U_{\text{max change}} = 1.6$ V at capacitor C2

A.3 (1.2 pt)

It's important to calculate $\int_{0V}^{6V} C(U)dU$, not just attempt to multiply $C(6\text{ V}) \cdot 6\text{ V}$

$$q_1 = 0.60\ \mu\text{C}; \quad q_2 = 1.3\ \mu\text{C}$$

Part B: Calibrating NCT thermistor (1 point)

B.1 (1.0 pt)

$R_0 = \frac{U_{T0}R_3}{U - U_{T0}} \mathrm{e}^{-B/T}$,

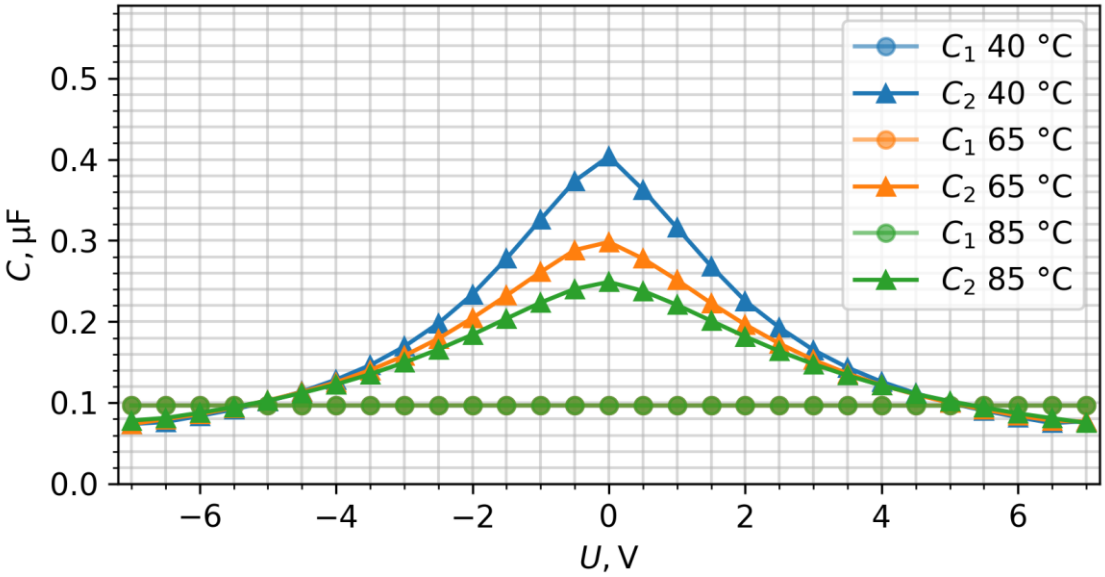
where $U = 3.3$ V, U_{T0} – uT at room temperature, T – room temperature in kelvins

$R_0 = 0.0341\ \Omega$.

Part C: Capacitors at different temperatures (3 points)

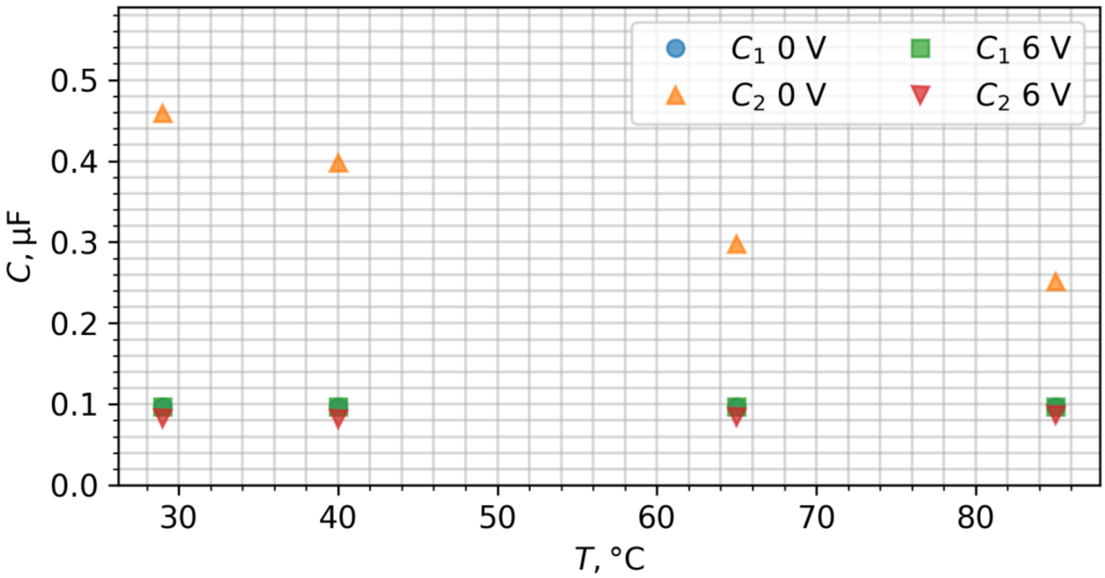
C.1 (1.3 pt)

Graphs $C_1(U,T)$ should always stay constant, $C_2(U)$ must be highest at $0\ \mathrm{V}$



C.2 (0.5 pt)

Graph $C_1(T)$ should always stay constant



C.3 (1.2 pt)

$C_1(85\ °C)/C_1(40\ °C)\ |_{0V} = 1.00$

$C_1(85\ °C)/C_1(40\ °C)\ |_{6V} = 1.00$

$C_2(85\ °C)/C_2(40\ °C)\ |_{0V} = 0.63$

$C_2(85\ °C)/C_2(40\ °C)\ |_{6V} = 1.06$

Part D: Sources of measurement errors (2 points)

D.1 (1.0 pt)

Initial settings:

S1 position	IN connection
C1	-9V or GND

Process:

Step number	S1 position	IN connection	Duration, s	Measured variable
1	C1	+9V	0.2 s (any short time is good)	
2	C1	Free		$\|du_C(t)\|/dt$
3	C1	+9V	5 s (has to be much longer than first)	
4	C1	Free		$\|du_C(t)\|/dt$

Verification: $|du_C(t)|/dt|_2 = |du_C(t)|/dt|_4$

Main source of error: 1 (Leakage current.)

D.2 (1.0 pt)

Initial settings:

S1 position	IN connection
C2	-9V or GND

Process:

Step number	S1 position	IN connection	Duration, s	Measured variable
1	C2	+9V	0.2 s (any short time is good)	
2	C2	Free		$\lvert \mathrm{d}uC(t)\rvert/\mathrm{d}t$
3	C2	+9V	5 s (has to be much longer than first)	
4	C2	Free		$\lvert \mathrm{d}uC(t)\rvert/\mathrm{d}t$

Verification: $\lvert \mathrm{d}uC(t)\rvert/\mathrm{d}t\rvert_2 \gg \lvert \mathrm{d}uC(t)\rvert/\mathrm{d}t\rvert_4$

Alternatively,

$$\frac{\lvert \mathrm{d}uC(t)\rvert/\mathrm{d}t\rvert_2}{\lvert \mathrm{d}uC(t)\rvert/\mathrm{d}t\rvert_4} > 2.$$

Main source of error: 2 (Polarization properties of the capacitor's dielectric media)

Light Emitting Diodes (LEDs) (10 points)

This experiment is designed to investigate the electrical and thermal properties of LEDs. For the temperature measurements of the PCB you should use coefficients, obtained in Experiment-1 B.1 section. The electric circuit used in this experiment is shown in Fig. 2.1. For equipment guide see description for question 1.

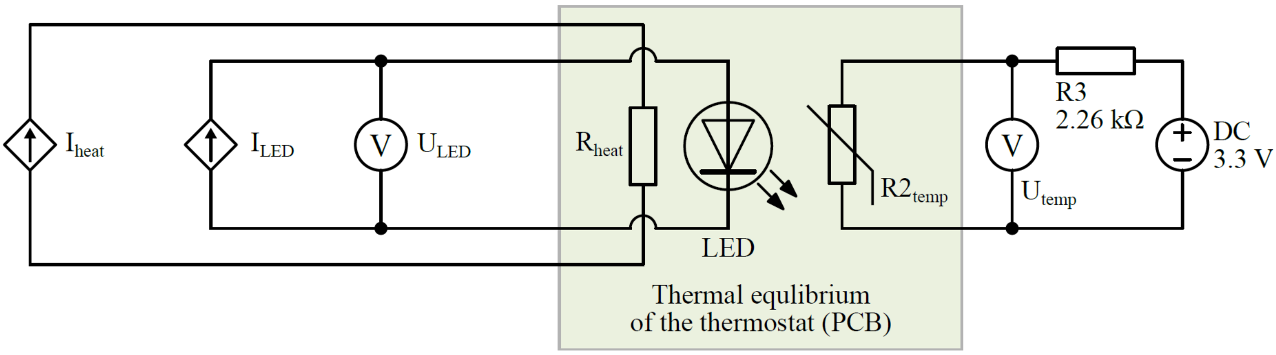


Figure 2.1. Experimental setup of the LED investigation experiment. LED is driven by the constant current (continuous or pulsed mode) and forward voltage measured by high impedance voltmeter. Heating and temperature measurement parts are the same as in Experiment 1. Thermal equilibrium is maintained between all components on printed circuit board (PCB).

The LEDs typically are driven by the constant current in contrast to constant voltage used for incandescent lamps. The measured voltage of the LEDs depends on set current and temperature of the semiconductor die. The mathematical expression of volt-ampere characteristics is complicated and depends on physical and technological parameters, which are usually not known. Therefore, in this experiment the two-dimensional dependence of the voltage vs LED current and LED die temperature T_J will have to be investigated:

$$U_{LED} = \text{function}(I_{LED},\ T_J).$$

The thermal resistance between the LED semiconductor die and the PCB is related to electrical power P as follows (at several values of the current (I_{LED})):

$$\frac{\Delta T}{P} = \frac{(T_J - T_{PCB})}{P}.$$

Caution: LED can be driven at continuous current or short current pulses. In the latter case it is assumed that the duration of the pulse is short enough to avoid the LED self-heating (for example 1 ms pulse duration with measurements spaced at least 100 ms apart), and to assume that $T_J = T_{PCB}$ at such driving regime. During the continuous operation $T_J > T_{PCB}$ and thermal resistance $\frac{\Delta T}{P}$ can be calculated.

Part A. Volt-ampere characteristics at different temperatures (5.0 points)

The physical mechanisms of the heating in both Experiment 1 and 2 are the same. Hence, you can use the result you obtained earlier in Experiment 1 to relate thermistor voltage with its temperature. Alternatively, you can use this explicit approximate relation:

$$T(U) = \frac{3500}{9.9 - \ln(\frac{1}{U} - 0.3)},$$

where T is temperature of the thermistor, expressed in kelvins, and U is voltage on the thermistor, expressed in volts.

Measure and graph the Current vs Voltage of the LED at temperatures ranging from room temperature to 80 °C in pulsed mode.

A.1	Measure and graph $I_{\text{LED\_pulsed}}(U_{\text{LED\_pulsed}}, T)$ dependence in the range from 3 mA to 50 mA at the room temperature, and 40, 60, and 80 °C. Draw all curves on the same graph.	2.5pt

A.2	In the answer sheet, fill the table with $U_{\text{LED\_pulsed}}$ values at 3, 10, 20, and 40 mA driving currents $I_{\text{LED\_pulsed}}$ at room temperature, 40, 60, and 80 °C.	1.0pt

A.3	Graph main points of $U_{\text{LED\_pulsed}}(I_{\text{LED\_pulsed}}, T)$ (those listed in question A.2) and calculate (approximate graphically) the linear voltage dependence on the temperature coefficient ($\Delta U(I)/\Delta T$) at 3, 10, 20, and 40 mA.	1.5pt

Part B. Measurement of the LED volt-ampere characteristics at continuous driving current (3.5 points)

B.1	Measure and graph the $I_{\text{LED\_continuous}}(U_{\text{LED\_continuous}})$ dependence in the range from 3 mA to 50 mA with the heater turned off in the continuous driving regime. In the answer sheet, also write down the values of $U_{\text{LED\_continuous}}$, PCB (thermostat) temperature T_{PCB}, and the difference $\Delta U = U_{\text{LED\_pulsed}} - U_{\text{LED\_continuous}}$ at 3, 10, 20, and 40 mA.	1.5pt

B.2	Since the resistance of the LEDs is not constant (depends on current), the term Dynamic Resistance is used and expressed as $\frac{\mathrm{d}U}{\mathrm{d}I}$. Using graph (B.1), estimate the reciprocal of the LED dynamic resistance $1/(\frac{\mathrm{d}U}{\mathrm{d}I}) = \frac{\mathrm{d}I}{\mathrm{d}U}$. In the answer sheet, write the values of $\frac{\mathrm{d}I}{\mathrm{d}U}$ at 3, 10, 20, and 40 mA. Draw tangents $\frac{\mathrm{d}I}{\mathrm{d}U}$ at these points on the graph.	0.5pt

B.3	Calculate and graph the difference $\Delta T(P)$between the temperature of continuously operating semiconductor die (T_{J}) and temperature of the PCB (T_{PCB}) as a function of electrical power (at 3, 10, 20, and 40 mA). Calculate (approximate graphically) the linear LED thermal resistance $\frac{\Delta T}{P}$, and write it in the answer sheet. *Note*: Assume that all electrical energy consumed by LED is converted into the heat and the energy emitted as light can be ignored.	1.5pt

Part C. Calculation of the LED current drift due to the temperature (1.5 points)

In the Introduction, it was mentioned that LEDs are typically driven by the constant current, but not constant voltage. Assume that one decided to drive the LED at nominal current value of 20 mA with constant voltage value you have measured for 20 mA current in the task B.1.

C.1	Using the LED characteristics calculated in section B, estimate the actual current flowing through LED, if voltage is kept constant (voltage measured in B.1, $U(20\mathrm{mA})$), but PCB temperature is at 0 °C and 40 °C.	1.5pt

Light Emitting Diodes (LEDs) (10 points)

Part A. Volt-ampere characteristics at different temperatures (5.0 points)

A.1 (2.5 pt)
Graph $I_{\text{LED\_pulsed}}(U_{\text{LED\_pulsed}}, T)$ dependence the room temperature, and 40, 60, and 80 °C (pulsed mode) on the millimeter paper:

A.2 (1.0 pt)
Fill the table with the corresponding values of $U_{\text{LED\_pulsed}}(I_{\text{LED\_pulsed}}, T)$:

	"Room" ______C	40 °C	60 °C	80 °C
3 mA				
10 mA				
20 mA				
40 mA				

A.3 (1.5 pt)
Graph main points of $U_{\text{LED\_pulsed}}(I_{\text{LED\_pulsed}}, T)$ (A.2) on the millimeter paper. Use linear graphical approximation to determine $\frac{\Delta U(I)}{\Delta T}$:

Linear graphical approximation:

I_{LED}	3 mA	10 mA	20 mA	40 mA
$\frac{\Delta U(I)}{\Delta T}$				

Part B. Measurement of the LED Volt-Ampere characteristics at continuous driving current (3.5 points)

B.1 (1.5 pt)
Graph $I_{\text{LED\_continuous}}(U_{\text{LED\_continuous}})$ dependence at room temperature (continuous mode) on the millimeter paper:

Fill the table with the corresponding values:

I_{LED}	3 mA	10 mA	20 mA	40 mA
U_{LED}				
ΔU				
T_{J}				
T_{PCB}				

B.2 (0.5 pt)

I_{LED}	3 mA	10 mA	20 mA	40 mA
$\frac{dI}{dU}$				

B.3 (1.5 pt)
Graph $\Delta T(P)$ on the millimeter paper:

Fill the table with the corresponding values:

I_{LED}	3 mA	10 mA	20 mA	40 mA
ΔT				

Linear LED thermal resistance $\frac{\Delta T}{P}$:

Part C. Calculation of the LED current drift due to the temperature (1.5 points)

C.1 (1.5 pt)

$I_{\mathrm{LED}}(U_{20\,\mathrm{mA}}, 0\,^{\circ}\mathrm{C}) =$

$I_{\mathrm{LED}}(U_{20\,\mathrm{mA}}, 40\,^{\circ}\mathrm{C}) =$

Light Emitting Diodes (LEDs)

Volt-Ampere characteristics of the LED have to be measured in two modes: pulsed (part A) and continuous (part B). Running LED continuously produces a noticeable amount of heat, while running it in the pulsed mode allows minimizing and neglecting self-heating effect.

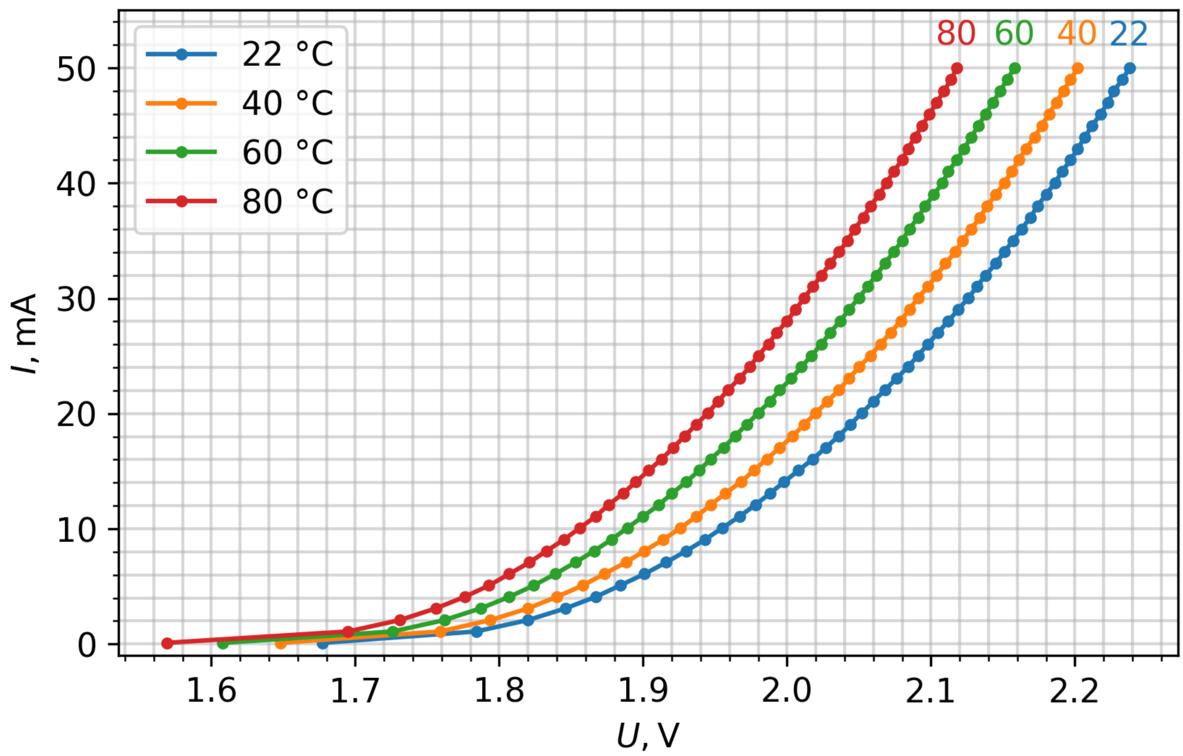
Students have to be able to run the automated $I_{\mathrm{LED}}(U_{\mathrm{LED}})$ measurement procedure and extract the required point by visually interpolating data for required values of I_{LED}.

The temperature of PCB is controlled by changing the current of the heating circuit. The heating and temperature measurement parts of this Experiment are identical to the Experiment 1.

Part A: Volt-ampere characteristics at different temperatures (5.0 points)

A.1 (2.5 pt.)

Graph $I_{\mathrm{LED}}(U_{\mathrm{LED}})$ has to be accurate (in right range) and smooth.



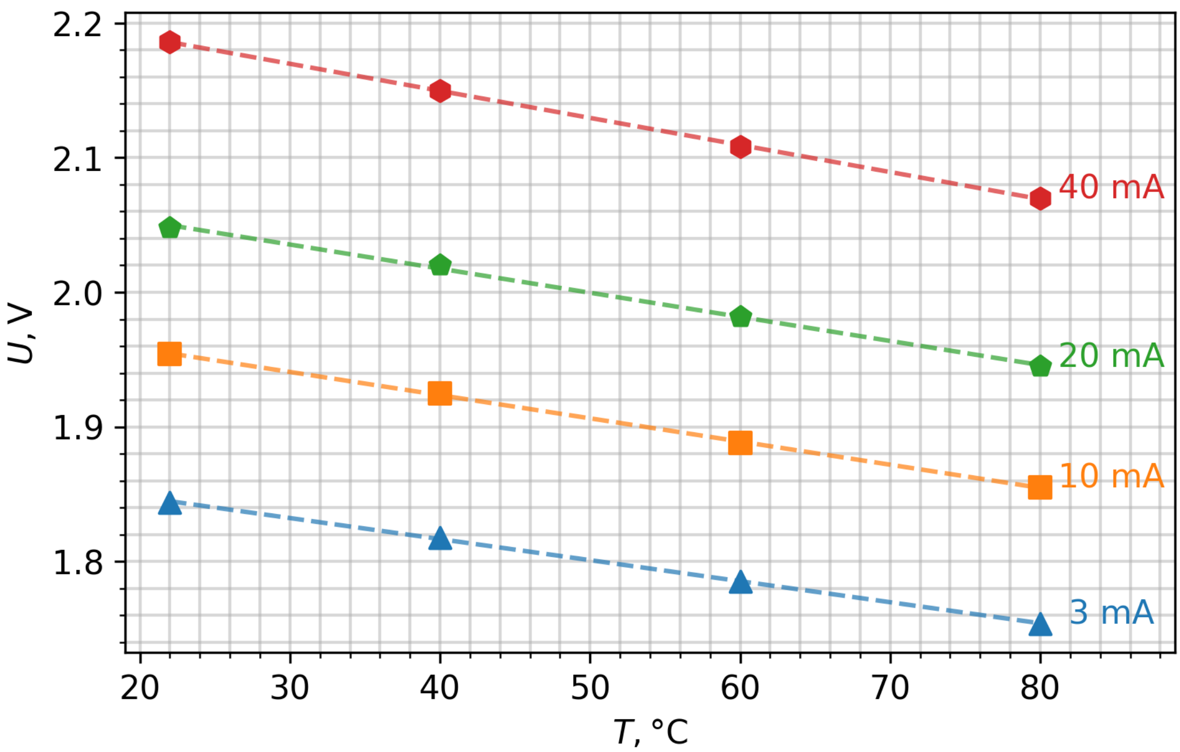
A.2 (1.0 pt.)

$U_{\text{LED}}(I_{\text{LED}}, T)$:

I_{LED} \ T	"Room" 22 °C	40 °C	60°C	80°C
3 mA	1.844 V	1.818	1.785	1.754
10 mA	1.954	1.925	1.888	1.855
20 mA	2.048	2.02	1.982	1.945
40 mA	2.186	2.15	2.108	2.07

A.3 (1.5 pt.)

Graphed $U_{\text{LED}}(I_{\text{LED}}, T)$ from A.2 data. $U_{\text{LED}}(T)$ should show clear linear trend and be approximated graphically. The slope $\left(\frac{\Delta U(I,T)}{\Delta T}\right)$ should also be calculated.

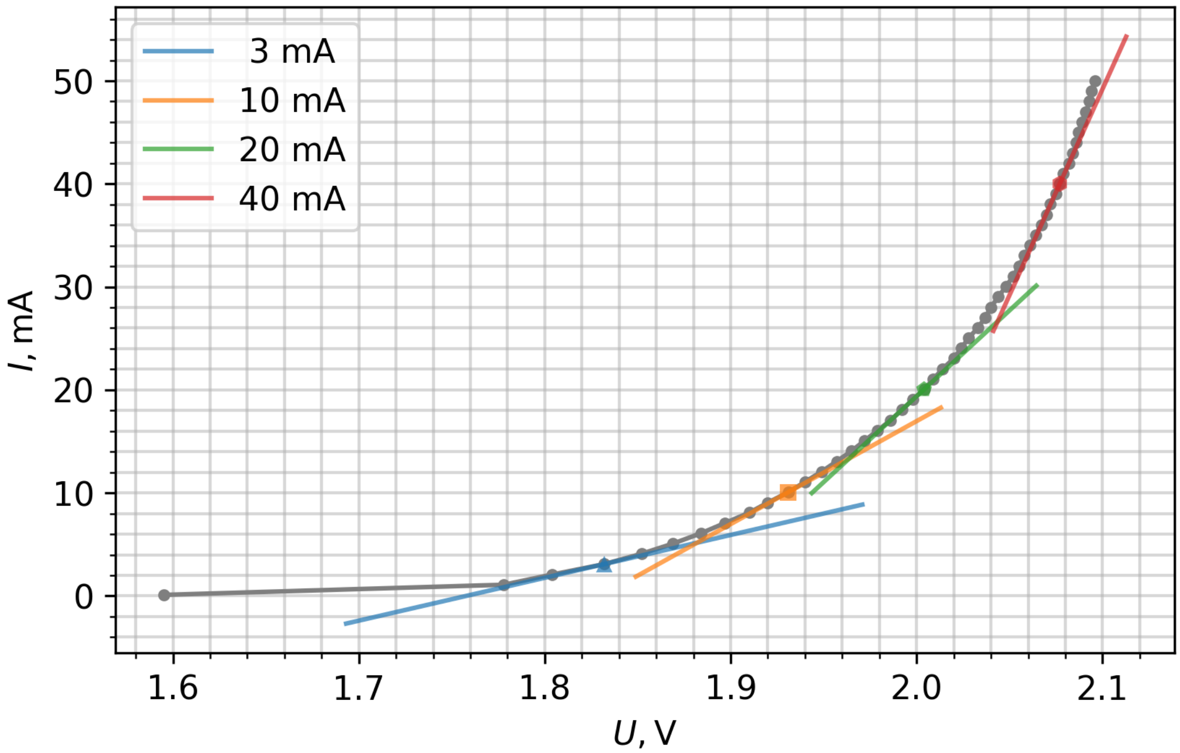


I_{LED}	3 mA	10 mA	20 mA	40 mA
$\left(\frac{\Delta U(I,T)}{\Delta T}\right)$	-1.55 mV/K	-1.7 mV/K	-1.8 mV/K	-2.0 mV/K

Part B: Measurement of the LED Volt-Ampere characteristics at continuous driving current (3.5 points)

B.1 (1.5 pt.)

Graph $I_{\text{LED}}(U_{\text{LED}})$ has to be accurate (in right range) and smooth.



I_{LED}	3 mA	10 mA	20 mA	40 mA
U_{LED}	1.83 V	1.93 V	2.00 V	2.08 V
ΔU	0.014 V	0.024 V	0.048 V	0.106 V
T_{J}	~ 32.3 °C	~43 °C	~49 °C	~76.5 °C
T_{PCB}	~ 25–30 °C	~ 30–35 °C	~ 33–37 °C	~ 35–40 °C

T_{J} are the most important parameters to be calculated in this section.
T_{J} have to be calculated by matching the U_{LED} values of B section with calibration curves in A section at certain current values. This can be done by selecting the nearest points of A.3 and calculating (interpolating) the T_{J} using the calculated $\left(\frac{\Delta U(I,T)}{\Delta T}\right)$ coefficients. Graphical interpolation is also possible, but is not as accurate as the first one.

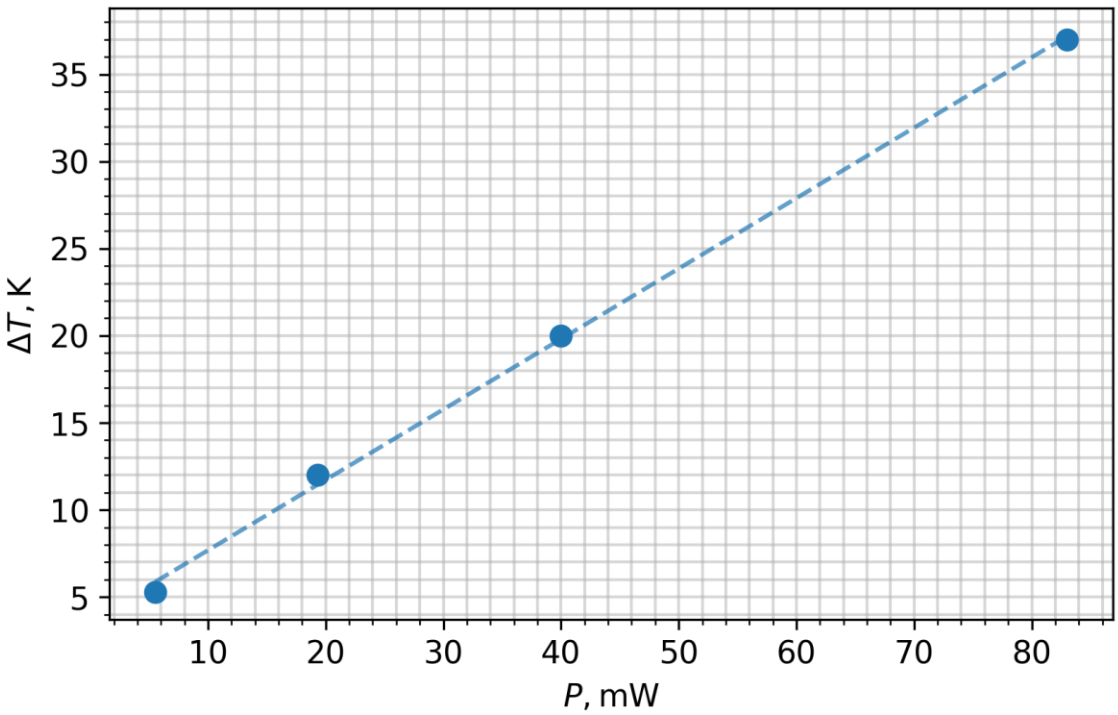
B.2 (0.5 pt.)

The dynamic resistance of the LED has to be calculated as derivative at the asked values of I_{LED}.

I_{LED}	3 mA	10 mA	20 mA	40 mA
$\frac{\mathrm{d}I}{\mathrm{d}U}$	41.6 mA/V	100 mA/V	166.7 mA/V	400 mA/V

B.3 (1.5 pt.)

Graphed $\Delta T(P)$.
ΔT for each I_{LED} has to be calculated as $\Delta T = T_{\mathrm{J}} - T_{\mathrm{PCB}}$ from the data of B.1.
Caution: during the measurement of B.1, the temperature of the PCB is not constant and rises up to ~7 °C above the "room" temperature at higher currents. This has to be taken into account when calculating ΔT.
The generated heat is taken as electrical power: $P = I_{\mathrm{LED}} \times U_{\mathrm{LED}}$. The energy emitted by the escaping light is neglected.
The graph should have a clear linear trend and approximated graphically. Thermal resistance is calculated as linear slope $\frac{\mathrm{d}}{\mathrm{d}P}\left(\Delta T(P)\right) \cong 400\ \mathrm{K/W}$.



I_{LED}	3 mA	10 mA	20 mA	40 mA
ΔT	5.0 K	12 K	20 K	37 K

Part C: Calculation of the LED current drift due to the temperature (1.5 points).

C.1 (1.5 pt)

Method 1:
The I_{LED} under constant $U_{\mathrm{LED}} = U_{20\mathrm{mA}}$ is calculated (estimated):

$$I_{\mathrm{LED}}(U_{20\mathrm{mA}}, T) = 20\ \mathrm{mA} - (T - T_{\mathrm{PCB}}) \times \left(\frac{\Delta U(20\ \mathrm{mA}, T)}{\Delta T}\right) \times \frac{\mathrm{d}I(20\ \mathrm{mA}, U)}{\mathrm{d}U}.$$

To be accurate, we have to understand, that $\frac{\mathrm{d}I(20\ \mathrm{mA}, U)}{\mathrm{d}U}$ from B.1 involves the temperature increase of the PCB with rising current, but there is no technical capabilities to perform the measurements at constant PCB temperature. Furthermore, since the current is decreasing significantly, application of the derivatives at 20 mA is not very accurate.

Method 2:
The required values can be calculated by interpolating/extrapolating A.3 data with T_{J} calculated using ΔT values from B.3. Accuracy in this way suffers a bit due to the dependence of electrical power P on temperature.

$I_{LED}(U_{20\mathrm{mA}}, 0°\mathrm{C}) \cong 10\text{–}15\ \mathrm{mA}$, $\quad I_{LED}(U_{20\mathrm{mA}}, 40°\mathrm{C}) \cong 22\ \mathrm{mA}$.

Planetary Physics (10 points)

This problem consists of two independent problems related to the interior of planets. The effects of the surface curvature of the planets can be neglected. You might need to use the formula

$$(1+x)^{\varepsilon} \approx 1+\varepsilon x, \text{ when } |x| \ll 1. \quad (1)$$

Part A. Mid-ocean ridge (5.0 points)

Consider a large vessel of water that is situated in a uniform gravitational field with free-fall acceleration g. Two vertical rectangular plates parallel to each other are fitted into the vessel so that the vertical edges of the plates are in a tight gap-less contact with the vertical walls of the vessel. Length h of each plate is immersed in water (Fig. 1). The width of the plates along the y-axis is w, water density is ρ_0.

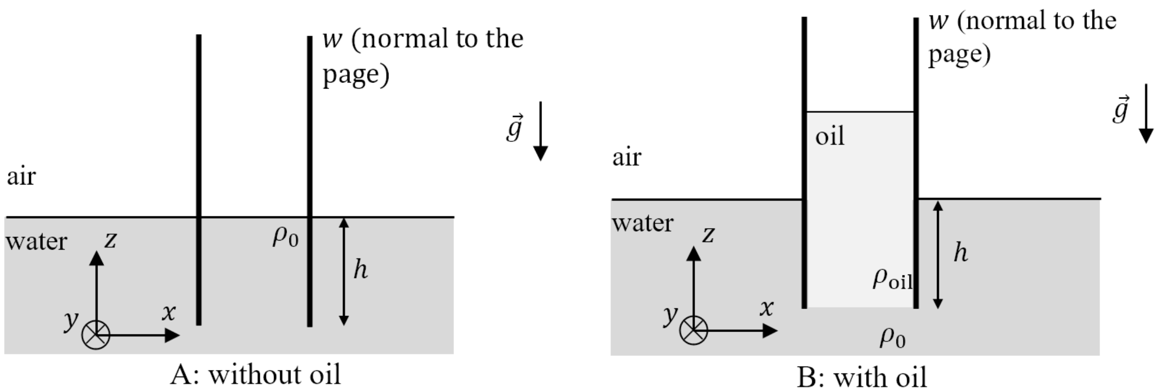


Figure 1. Parallel plates in water.

Oil of density ρ_{oil} ($\rho_{oil} < \rho_0$) is poured into the space between the plates until the lower level of the oil has reached the lower edges of the plates. Assume that plates and vessel edges are high enough for oil not to overflow them. Surface tension and mixing of fluids can be neglected.

A.1	What is the x-component of the net force F_x acting on the right plate (magnitude and direction)?	0.8pt

Fig. 2 shows a cross-section of a mid-ocean ridge. It consists of overlaying layers of mantle, crust and ocean water. The mantle is composed of rocks that we assume can flow in geological timescales and therefore, in this problem will be treated as a fluid. The thickness of the crust is much smaller than the characteristic length scale in the x-direction, hence, the crust behaves as a freely bendable plate. To high accuracy, such a ridge can be modeled as a two-dimensional system, without any variation of variables along the y-axis, which is perpendicular to the plane of Fig. 2. Assume that the ridge length L along the y-axis is much larger than any other length introduced in this problem.

At the centre of the ridge the thickness of the crust is assumed to be zero. As the horizontal distance x from the centre increases, the crust gets thicker and approaches a constant thickness D as $x \to \infty$. Correspondingly, the ocean floor subsides by a vertical height h below the top of the ridge O, which we define as the origin of our coordinate system (see Fig. 2). Water density ρ_0 and temperature T_0 can be assumed to be constant in space and time. The same can be assumed for mantle density ρ_1 and its temperature T_1. The temperature of the crust T is also constant in time but can depend on position.

It is known that, to high accuracy, the crustal material expands linearly with temperature T. Since water and mantle temperatures are assumed to be constant, it is convenient to use a rescaled version of the thermal expansion coefficient. Then $l(T) = l_1\left[1 - k_l(T_1 - T)/(T_1 - T_0)\right]$, where l is the length of a piece of crustal material, l_1 is its length at temperature T_1, and k_l is the rescaled thermal expansion coefficient, which can be assumed to be constant.

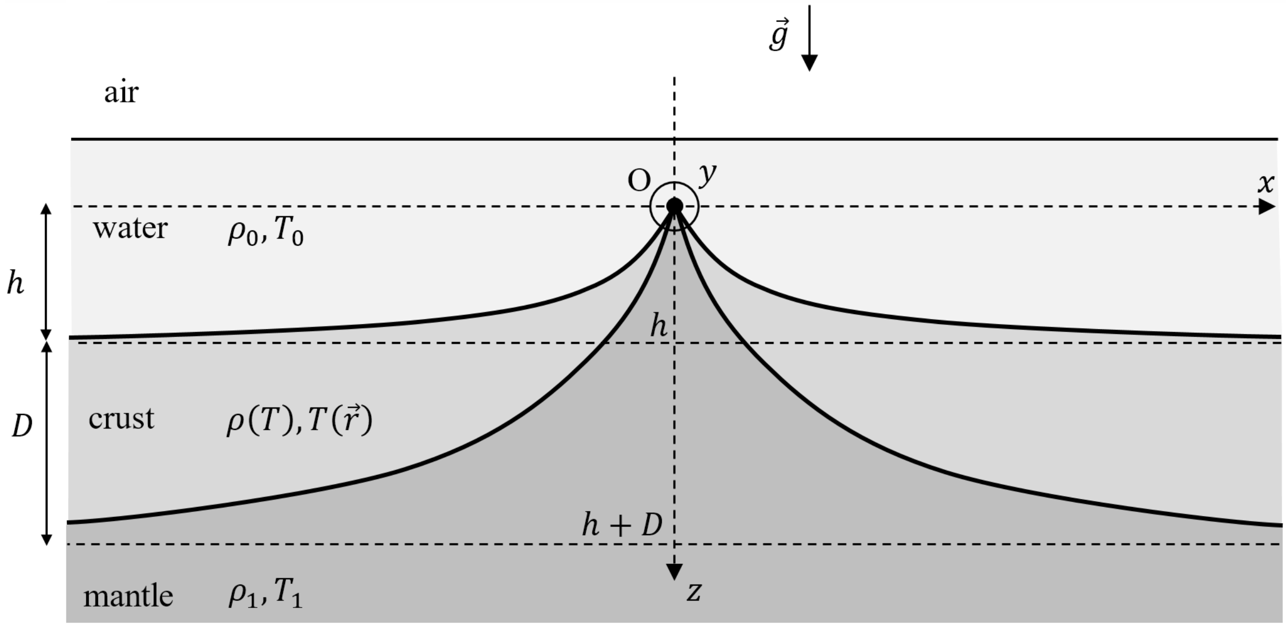


Figure 2. Mid-ocean ridge. Note that the z-axis is pointing downwards.

A.2 Assuming that the crust is isotropic, find how its density ρ depends on its temperature T. Assuming that $|k_l| \ll 1$, write your answer in the approximate form **0.6pt**

$$\rho(T) \approx \rho_1\left[1 + k\frac{T_1 - T}{T_1 - T_0}\right], \quad (2)$$

where terms of order k_l^2 and higher are neglected. Then, identify constant k.

It is known that $k > 0$. Also, thermal conductivity of the crust κ can be assumed to be constant. As a consequence, very far away from the ridge axis the temperature of the crust depends linearly with depth.

A.3 By assuming that mantle and water each behave like an incompressible fluid at hydrostatic equilibrium, express the far-distance crust thickness D in terms of h, ρ_0, ρ_1, and k. Any motion of the material can be neglected. **1.1pt**

A.4 Find, to the leading order in k, the net horizontal force F acting on the right half ($x > 0$) of the crust in terms of ρ_0, ρ_1, h, L, k and g. **1.6pt**

Suppose that crust is thermally isolated from the rest of the Earth. As a result of heat conduction, the temperatures of the upper and lower surfaces of the crust are going to get closer to each other until the crust reaches thermal equilibrium. Specific heat of the crust is c and can be assumed to be constant.

A.5	By using dimensional analysis or order-of-magnitude analysis, estimate the characteristic time τ in which the difference between the upper and lower surface temperatures of the crust far away from the ridge axis is going to approach zero. You can assume that τ does not depend on the two initial surface temperatures of the crust.	0.9pt

Part B. Seismic waves in a stratified medium (5.0 points)

Suppose that a short earthquake happens at the surface of some planet. The seismic waves can be assumed to originate from a line source situated at $z = x = 0$, where x is the horizontal coordinate and z is the depth below the surface (Fig. 3). The seismic wave source can be assumed to be much longer than any other length considered in this question.

As a result of the earthquake, a uniform flux of the so-called longitudinal P waves is emitted along all the directions in the x-z plane that have positive component along the z-axis. Since the wave theory in a solid is generally complicated, in this problem we neglect all the other waves emitted by the earthquake. The crust of the planet is stratified so that the P-wave speed v depends on depth z according to $v = v_0(1+z/z_0)$, where v_0 is the speed at the surface and z_0 is a known positive constant.

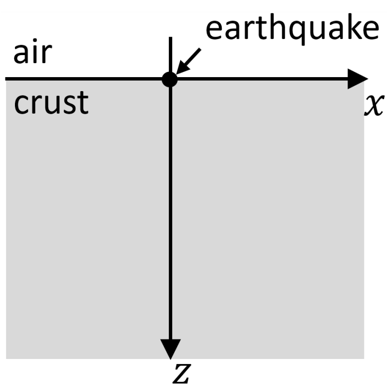


Figure 3. Coordinate system used in part B.

B.1	Consider a single ray emitted by the earthquake that makes an initial angle $0 < \theta_0 < \pi/2$ with the z-axis and travels in the x-z plane. What is the horizontal coordinate $x_1(\theta_0) \neq 0$ at which this ray can be detected at the surface of the planet? It is known that the ray path is an arc of a circle. Write your answer in the form $x_1(\theta_0) = A\cot(b\theta_0)$, where A and b are constants to be found.	1.5pt

If you were unable to find A and b, in the following questions you can use the result $x_1(\theta_0) = A\cot(b\theta_0)$ as given. Suppose that total energy per unit length of the source released as P waves into the crust during the earthquake is E. Assume that waves are completely absorbed when they reach the surface of the planet from below.

B.2	Find how the energy density per unit area $\varepsilon(x)$ absorbed by the surface depends on the distance along the surface x. Sketch the plot of $\varepsilon(x)$.	1.5pt

From now on, assume that the waves are instead fully reflected when reaching the surface. Imagine a device positioned at $z = x = 0$ that has the same geometry as the previously considered earthquake source. The device is capable of emitting P waves in a freely chosen angular distribution. We make the device emit a signal with a narrow range of emission angles. In particular, the initial angle the signal makes with the vertical belongs to the interval $[\theta_0 - \frac{1}{2}\delta\theta_0, \theta_0 + \frac{1}{2}\delta\theta_0]$, where $0 < \theta_0 < \pi/2$, $\delta\theta_0 \ll 1$ and $\delta\theta_0 \ll \theta_0$.

B.3	At what distance x_{max} along the surface from the source is the furthest point that the signal does not reach? Write your answer in terms of θ_0, $\delta\theta_0$ and other constants given above.	2.0pt

Planetary Physics (10 points)

Part A. Mid-ocean ridge (5.0 points)

A.1 (0.8 pt)

$F_x =$

This force acts on the right plate to the (check): ☐ left ☐ right.

A.2 (0.6 pt)

$\rho(T) \approx$

$k =$

A.3 (1.1 pt)

$D =$

A.4 (1.6 pt)

$F \approx$

A.5 (0.9 pt)

$\tau =$

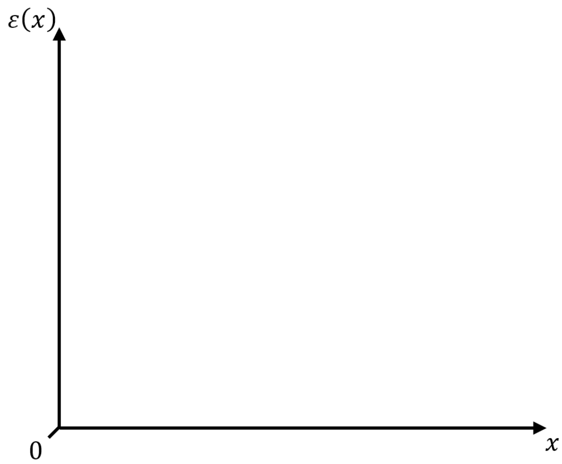
Part B. Seismic waves in a stratified medium (5.0 points)

B.1 (1.5 pt)

$x_1(\theta_0) =$

B.2 (1.5 pt)

$\varepsilon(x) =$



B.3 (2.0 pt)

$x_{\max} =$

Planetary Physics (10 points)

Part A. Mid-ocean ridge (5.0 points)

A.1 (0.8 points)

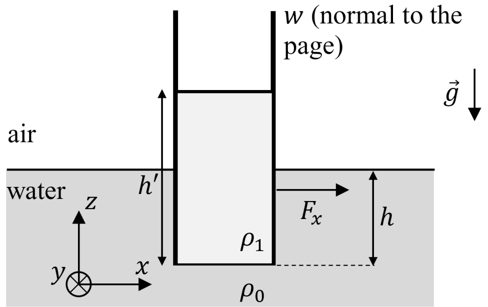


Figure 1

Let h' be the height of the column of oil (see Fig. 1). Then pressure at depth h below the water surface must be $p_h = \rho_0 g h = \rho_{\text{oil}} g h'$, from where $h' = \frac{\rho_0}{\rho_{\text{oil}}} h$. Horizontal force on the plate $F_x = F_1 - F_0$, where the force due to new fluid is $F_1 = \frac{\rho_{\text{oil}} g h'}{2} \cdot h' w$ and the force due to water is $F_0 = \frac{\rho_0 g h}{2} \cdot h w$.

Combining all the equation above, we get

$$F_x = \left(\frac{\rho_0}{\rho_{\text{oil}}} - 1\right) \frac{\rho_0 g h^2 w}{2}.$$

This force acts on the right plate to the right.

A.2 (0.6 points)

Consider a rectangular mass element of the crust. Since relation $l(T) = l_1 \left[1 - k_l \left(T_1 - T\right)/(T_1 - T_0)\right]$ holds for all three dimensions of the solid, its volume V satisfies

$$V = V_1 \left(1 - k_l \frac{T_1 - T}{T_1 - T_0}\right)^3,$$

where V_1 is the volume at $T = T_1$. If the mass of the element is m, density is then

$$\rho(T) = \frac{m}{V} = \frac{m}{V_1} \left(1 - k_l \frac{T_1 - T}{T_1 - T_0}\right)^{-3} = \rho_1 \left(1 - k_l \frac{T_1 - T}{T_1 - T_0}\right)^{-3}.$$

Since $k_l \ll 1$, this can be approximated as

$$\rho\left(T\right) \approx \rho_1 \left(1 + 3k_l \frac{T_1 - T}{T_1 - T_0}\right),$$

so that $k = 3k_l$.

A.3 (1.1 points)

Since mantle behaves like a fluid in hydrostatic equilibrium, pressure $p(x,z)$ at $z = h + D$ must be the same for all x. Therefore,

$$p(0, h+D) = p(\infty, h+D).$$

Similarly, we must have

$$p(0,0) = p(\infty, 0).$$

Hence, the change in pressure between $z = 0$ and $z = \infty$ must be the same at both $x = 0$ and $x = \infty$. At the ridge axis

$$p(0, h+D) - p(0,0) = \rho_1 g (h+D),$$

while far away

$$p(\infty, h+D) - p(\infty, 0) = \rho_0 g h + \int_h^{h+D} \rho(T(\infty, z))\, g\, \mathrm{d}z.$$

Since the temperature of the crust at $x = \infty$ depends linearly on height, after applying the relevant temperature boundary conditions,

$$T(\infty, z) = T_0 + (T_1 - T_0)\frac{z-h}{D}.$$

From all the equations above and by using the density formula given in the problem text,

$$\rho_1 g(h+D) = \rho_0 g h + \int_h^{h+D} \rho_1 \left(1 + k\frac{T_1 - T_0 - (T_1 - T_0)\frac{z-h}{D}}{T_1 - T_0}\right) g\, \mathrm{d}z,$$

from where we straightforwardly obtain

$$D = \frac{2}{k}\left(1 - \frac{\rho_0}{\rho_1}\right) h.$$

A.4 (1.6 points)

The net horizontal force on the half of the ridge is the difference between the pressure forces acting at $x = 0$ and $x = \infty$:

$$F = L\int_0^{h+D} p(0,z)\,\mathrm{d}z - L\int_0^{h} p(\infty, z).$$

From considerations of the previous question, pressure at $x = 0$ is

$$p(0,z) = p(0,0) + \rho_1 g z,$$

while very far away

$$p(\infty, z) = p(\infty, 0) + \rho_0 g z$$

The equations above can be combined into

$$F = L\int_0^{h+D} (p(0,0) + \rho_1 g z)\,\mathrm{d}z - L\int_0^{h} (p(\infty,0) + \rho_0 g z)\,\mathrm{d}z.$$

After a straightforward integration and using $p(0,0) = p(\infty, 0)$,

$$F = Lp(0,0)\,D + L\rho_1 g\frac{(h+D)^2}{2} - L\rho_0 g\frac{h^2}{2}.$$

Since $k \ll 1$, and $D \propto k^{-1}$, the term with $D^2 \propto k^{-2}$ is of the leading order, hence, after substituting the result of A.3, the required answer is

$$F \approx \frac{2gLh^2(\rho_1-\rho_0)^2}{k^2\rho_1}.$$

A.5 (0.9 points)

Method 1: dimensional analysis. The timescale τ is expected to depend only on density of the crust ρ_1, its specific heat c, thermal conductivity κ and thickness D. Hence, we can write

$$\tau = A\rho_1^\alpha c^\beta \kappa^\gamma D^\delta,$$

where A is a dimensionless constant. We will obtain the powers α–δ via dimensional analysis.

Define the symbols for different dimensions: L for length, M for mass, T for time and Θ for temperature. Then τ, ρ_1, c, κ and D have dimensions T, ML^{-3}, $\mathrm{L}^2\mathrm{T}^{-2}\Theta^{-1}$, $\mathrm{MLT}^{-3}\Theta^{-1}$ and L, respectively. The resulting set of linear equations to balance the powers of length, mass, time and temperature, respectively, is

$$\begin{cases} 0 = -3\alpha + 2\beta + \gamma + \delta, \\ 0 = \alpha + \gamma, \\ 1 = -2\beta - 3\gamma, \\ 0 = -\beta - \gamma. \end{cases}$$

This gives $\alpha = \beta = 1$, $\gamma = -1$, $\delta = 2$. Hence,

$$\tau = A\frac{c\rho_1 D^2}{\kappa}.$$

Method 2: scale analysis. Consider a piece of crust of area S. Heat flux that has to be transmitted through the crust is of order $Q \sim c\rho_1 SD\Delta T$, where $\Delta T = T_1 - T_0$. On the other hand, the law of thermal conductivity gives that $\kappa\frac{\Delta T}{D} \sim \frac{Q}{S\tau}$.

From the two equations, $c\rho_1 SD\Delta T \sim S\tau\kappa\frac{\Delta T}{D}$, from where we get that τ is independent of ΔT and

$$\tau \sim \frac{c\rho_1 D^2}{\kappa}.$$

Part B. Seismic waves in a stratified medium (5.0 points)

B.1 (1.5 points)

Seismic waves in this problem can be treated by using ray theory. Namely, their propagation is described by the Snell's law of refraction

$$n(0)\sin\theta_0 = n(z)\sin\theta,$$

where the refractive index is

$$n(z) = \frac{c}{v(z)} = \frac{c}{v_0\left(1+\frac{z}{z_0}\right)}$$

and c denotes the seismic wave speed in a material with refractive index $n = 1$. From the two equations above we have

$$v_0\left(1+\frac{z}{z_0}\right)\sin\theta_0 = v_0\sin\theta.$$

Method 1. Since this describes an arc of a circle, we have that at $\theta = \frac{\pi}{2}$, $z = R - R\sin\theta_0$ (Fig. 2), giving

$$\left(1+\frac{R-R\sin\theta_0}{z_0}\right)\sin\theta_0 = 1,$$

from where the circle radius $R = \frac{z_0}{\sin\theta_0}$. From simple geometry we get

$$x_1(\theta_0) = 2R\cos\theta_0,$$

leading to

$$x_1(\theta_0) = 2z_0\cot\theta_0,$$

i.e. $A = 2z_0$ and $b = 1$.

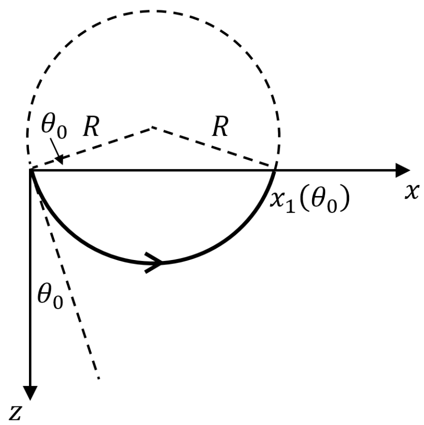


Figure 2

Method 2. Implicitly differentiating $v_0\left(1+\frac{z}{z_0}\right)\sin\theta_0 = v_0\sin\theta$ gives

$$\frac{\mathrm{d}z}{z_0}\sin\theta_0 = \cos\theta\,\mathrm{d}\theta.$$

An infinitesimal ray path length $\mathrm{d}l$ is related to the change in the vertical coordinate via

$$\mathrm{d}z = \mathrm{d}l\,\cos\theta,$$

giving

$$\mathrm{d}l = \frac{z_0}{\sin\theta_0}\,\mathrm{d}\theta.$$

This is an equation of an arc of a circle of radius $R = \frac{z_0}{\sin\theta_0}$

Alternatively, instead of considering an infinitesimal ray path length $\mathrm{d}l$, one can obtain the answer by writing

$$\cot\theta = \frac{\mathrm{d}z}{\mathrm{d}x} = \frac{\mathrm{d}z}{\mathrm{d}\theta}\frac{\mathrm{d}\theta}{\mathrm{d}x}.$$

The first derivative can be eliminated via Snell's law, leading to

$$\cot\theta = \frac{z_0\cos\theta}{\sin\theta_0}\frac{\mathrm{d}\theta}{\mathrm{d}x},$$

which can be integrated to get

$$x_1 = -\frac{z_0}{\sin\theta_0}\int_{\text{start}}^{\text{end}}\mathrm{d}\cos\theta = \frac{2z_0\cos\theta_0}{\sin\theta_0},$$

where we used Snell's law again to get that the ray has $\cos\theta = -\cos\theta_0$ at the point where it reaches the surface.

B.2 (1.5 points)

In two dimensions, $\frac{E}{\pi}\,\mathrm{d}\theta_0$ is the energy carried by rays that are emitted within interval $[\theta_0, \theta_0 + \mathrm{d}\theta_0)$. On the other hand, the energy carried by rays that arrive at $[x, x+\mathrm{d}x)$ is $\varepsilon\,\mathrm{d}x$. Therefore,

$$\varepsilon = \frac{E}{\pi}\left|\frac{\mathrm{d}\theta_0}{\mathrm{d}x}\right|.$$

Using the result of question B.1,

$$\frac{\mathrm{d}x}{\mathrm{d}\theta_0} = -\frac{Ab}{\sin^2(b\theta_0)} = -Ab\left(1+\cot^2(b\theta_0)\right) = -\frac{b\left(A^2+x^2\right)}{A}.$$

Hence,

$$\varepsilon(x) = \frac{EA}{\pi b\left(A^2+x^2\right)} = \frac{2Ez_0}{\pi\left(4z_0^2+x^2\right)}.$$

This function is plotted in Fig. 3.

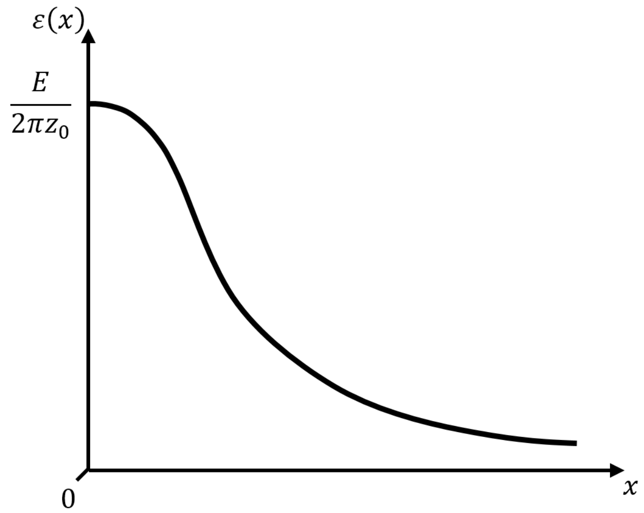


Figure 3. Plot of the function $\varepsilon(x)$.

B.3 (2.0 points)

Define $x_- = x_1\left(\theta_0 - \frac{\delta\theta_0}{2}\right)$ and $x_+ = x_1\left(\theta_0 + \frac{\delta\theta_0}{2}\right)$. To the leading order in $\delta\theta_0$, $x_- \approx x_+ \approx x_1(\theta_0)$. With each reflection of the signal, the horizontal distance between the points where the edges of the signal reflect increases by $|x_+ - x_-| = x_- - x_+$. When moving along the positive x-axis, these zones get wider until they overlap. If this happens after N reflections, then

$$N \approx \frac{x_1(\theta_0)}{x_- - x_+},$$

where the approximate sign tends to equality as $\delta\theta_0 \longrightarrow 0$.

The position where the zones start to overlap is at $x_{\max} = N x_1(\theta_0)$. Therefore,

$$x_{\max} = \frac{x_1(\theta_0)^2}{x_1\left(\theta_0 - \frac{\delta\theta_0}{2}\right) - x_1\left(\theta_0 + \frac{\delta\theta_0}{2}\right)}.$$

Since $\delta\theta_0 \ll \theta_0$, we can approximate

$$x_1\left(\theta_0 - \frac{\delta\theta_0}{2}\right) - x_1\left(\theta_0 + \frac{\delta\theta_0}{2}\right) \approx -\frac{\mathrm{d}x_1(\theta_0)}{\mathrm{d}\theta_0}\delta\theta_0 = \frac{Ab}{\sin^2(b\theta_0)}\delta\theta_0.$$

Combining the last two equations and substituting the $x_1(\theta_0)$ expression gives

$$x_{\max} = \frac{A\cos^2(b\theta_0)}{b\,\delta\theta_0} = \frac{2z_0\cos^2\theta_0}{\delta\theta_0}.$$

Electrostatic lens (10 points)

Consider a uniformly charged metallic ring of radius R and total charge q. The ring is a hollow toroid of thickness $2a \ll R$. This thickness can be neglected in parts A, B, C, and E. The xy plane coincides with the plane of the ring, while the z-axis is perpendicular to it, as shown in Figure 1. In parts A and B you might need to use the formula (Taylor expansion)

$$(1+x)^{\varepsilon} \approx 1 + \varepsilon x + \frac{1}{2}\varepsilon(\varepsilon-1)x^2, \text{ when } |x| \ll 1.$$

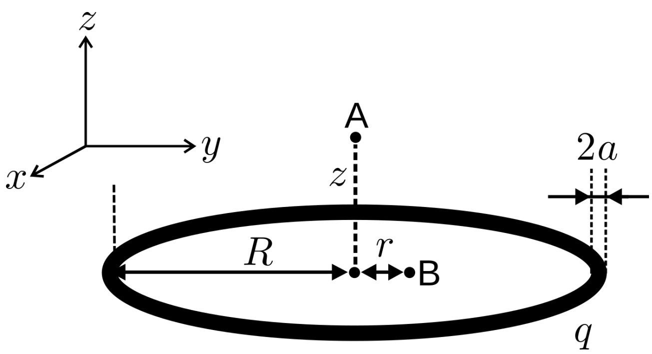


Figure 1. A charged ring of radius R.

Part A. Electrostatic potential on the axis of the ring (1 point)

A.1	Calculate the electrostatic potential $\Phi(z)$ along the axis of the ring at a z distance from its center (point A in Figure 1).	0.3pt

A.2	Calculate the electrostatic potential $\Phi(z)$ to the lowest non-zero power of z, assuming $z \ll R$.	0.4pt

A.3	An electron (mass m and charge $-e$) is placed at point A (Figure 1, $z \ll R$). What is the force acting on the electron? Looking at the expression of the force, determine the sign of q so that the resulting motion would correspond to oscillations. The moving electron does not influence the charge distribution on the ring.	0.2pt

A.4	What is the angular frequency ω of such harmonic oscillations?	0.1pt

Part B. Electrostatic potential in the plane of the ring (1.7 points)

In this part of the problem you will have to analyze the potential $\Phi(r)$ in the plane of the ring ($z = 0$) for $r \ll R$ (point B in Figure 1). To the lowest non-zero power of r the electrostatic potential is given by $\Phi(r) \approx q(\alpha + \beta r^2)$.

B.1	Find the expression for β. You might need to use the Taylor expansion formula given above.	1.5pt

B.2	An electron is placed at point B (Figure 1, $r \ll R$). What is the force acting on the electron? Looking at the expression of the force, determine the sign of q so that the resulting motion would correspond to harmonic oscillations. The moving electron does not influence the charge distribution on the ring.	0.2pt

Part C. The focal length of the idealized electrostatic lens: instantaneous charging (2.3 points)

One wants to build a device to focus electrons—an electrostatic lens. Let us consider the following construction. The ring is situated perpendicularly to the z-axis, as shown in Figure 2. We have a source that produces on-demand packets of non-relativistic electrons. Kinetic energy of these electrons is $E = mv^2/2$ (v is velocity) and they leave the source at precisely controlled moments. The system is programmed so that the ring is charge-neutral most of the time, but its charge becomes q when electrons are closer than a distance $d/2$ ($d \ll R$) from the plane of the ring (shaded region in Figure 2, called "active region"). In part C assume that charging and de-charging processes are instantaneous and the electric field "fills the space" instantaneously as well. One can neglect magnetic fields and assume that the velocity of electrons in the z-direction is constant. Moving electrons do not perturb the charge distribution on the ring.

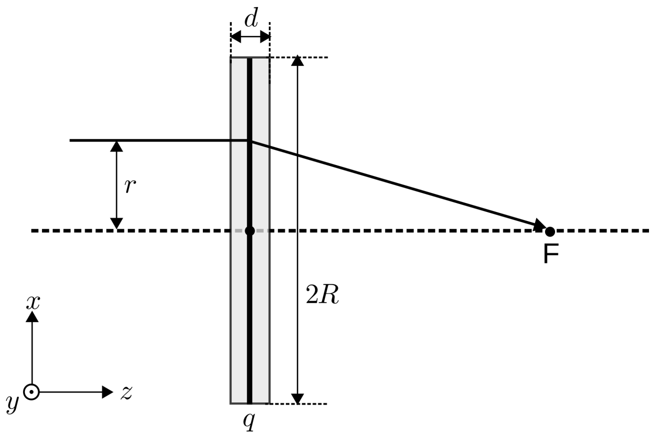


Figure 2. A model of an electrostatic lens.

C.1	Determine the focal length f of this lens. Assume that $f \gg d$. Express your answer in terms of the constant β from question B.1 and other known quantities. Assume that before reaching the "active region" the electron packet is parallel to the z-axis and $r \ll R$. The sign of q is such so that the lens is focusing.	1.3pt

In reality the electron source is placed on the z-axis at a distance $b > f$ from the center of the ring. Consider that electrons are no longer parallel to the z-axis before reaching the "active region", but are emitted from a point source at a range of different angles $\gamma \ll 1$ rad to the z-axis. Electrons are focused in a point situated at a distance c from the center of the ring.

C.2	Find c. Express your answer in terms of the constant β from question B.1 and other known quantities.	0.8pt

C.3	Is the equation of a thin optical lens $$\frac{1}{b}+\frac{1}{c}=\frac{1}{f}$$ fulfilled for the electrostatic lens? Show it by explicitly calculating $1/b+1/c$.	0.2pt

Part D. The ring as a capacitor (3 points)

The model considered above was idealized and we assumed that the ring charged instantaneously. In reality charging is non-instantaneous, as the ring is a capacitor with a finite capacitance C. In this part we will analyze the properties of this capacitor. You might need the following integrals:

$$\int \frac{\mathrm{d}x}{\sin x} = -\ln\left|\frac{\cos x + 1}{\sin x}\right| + \mathrm{const}$$

and

$$\int \frac{\mathrm{d}x}{\sqrt{1+x^2}} = \ln\left|x+\sqrt{1+x^2}\right| + \mathrm{const}.$$

D.1	Calculate the capacitance C of the ring. Consider that the ring has a finite width $2a$, but remember that $a \ll R$.	2.0pt

When electrons reach the "active region", the ring is connected to a source of voltage V_0 (Figure 3). When electrons pass the "active region", the ring is connected to the ground. The resistance of contacts is R_0 and the resistance of the ring itself can be neglected.

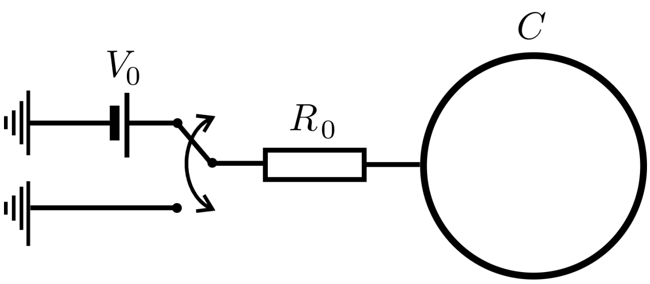


Figure 3. Charging of the electrostatic lens.

D.2	Determine the dependence of the charge on the ring as a function of time, $q(t)$, and make a schematic plot of this dependence. $t = 0$ corresponds to a time moment when electrons are in the plane of the ring. What is the charge on the ring q_0 when the absolute value $q(t)$ is maximal? The capacitance of the ring is C (i.e., you do not have to use the actual expression found in D.1). *Remark*: the drawn polarity in Figure 3 is for indicative purposes only. The sign should be chosen so that the lens is focusing.	1.0pt

Part E. Focal length of a more realistic lens: non-instantaneous charging (2 points)

In this part of the problem, we will consider the action of this more realistic lens. Here we will again neglect the width of the ring $2a$ and will assume that electrons travel parallel to the z-axis before reaching the ”active region”. However, the charging of the ring is no longer instantaneous.

E.1	Find the focal length f of the lens. Assume that $f/v \gg R_0C$, but d/v and R_0C are of the same order of magnitude. Express your answer in terms of the constant β from part B and other known quantities.	1.7pt

E.2	You will see, that the result for f is similar to that obtained in part C, whereby the value q is substituted with q_{eff}. Find the expression for q_{eff} in terms of quantities given in formulation of the problem.	0.3pt

Electrostatic lens (10 points)

Part A. Electrostatic potential on the axis of the ring (1 point)

A.1 (0.3 pt)

$\Phi(z) =$

A.2 (0.4 pt)

$\Phi(z) \approx$

A.3 (0.2 pt)

$F(z) =$

Circle the right answer: $q < 0$ *or* $q > 0$.

A.4 (0.1 pt)

$\omega =$

Part B. Electrostatic potential in the plane of the ring (1.7 points)

B.1 (1.5 pt)

$\beta =$

B.2 (0.2 pt)

$F(r) =$

Circle the right answer: $q < 0$ *or* $q > 0$.

Part C. The focal length of the idealized electrostatic lens: instantaneous charging (2.3 points)

C.1 (1.3 pt)

$f =$

C.2 (0.8 pt)

$c =$

C.3 (0.2 pt)

Circle the right answer. The equation of the thin optical lens is: valid *or* not valid.

Part D. The ring as a capacitor (3 points)

D.1 (2.0 pt)

$C =$

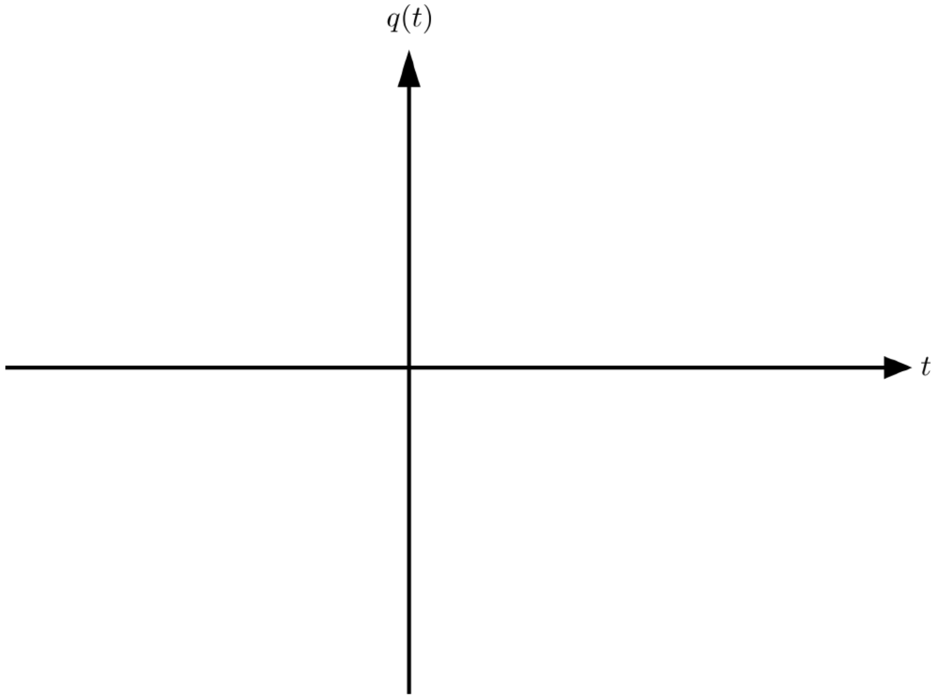
D.2 (1.0 pt)

For $-\frac{d}{2v} < t < \frac{d}{2v}$, $q(t) =$

For $t > \frac{d}{2v}$, $q(t) =$

$q_0 =$

Charge on the ring as a function of time:



Part E. Focal length of a more realistic lens: non-instantaneous charging (2 points)

E.1 (1.7 pt)

$f =$

E.2 (0.3 pt)

$q_{\text{eff}} =$

Electrostatic lens (10 points)

Part A. Electrostatic potential on the axis of the ring (1 point)

A.1 (0.3 points)

The linear charge density of the ring is $\lambda = q/(2\pi R)$. All the points of the ring are situated a distance $\sqrt{R^2+z^2}$ away from point A. Integrating over the whole ring we readily obtain:

$$\Phi(z) = \frac{q}{4\pi\varepsilon_0}\frac{1}{\sqrt{R^2+z^2}}.$$

A.2 (0.4 points)

Using an expansion in powers of z we obtain:

$$\Phi(z) = \frac{q}{4\pi\varepsilon_0}\frac{1}{\sqrt{R^2+z^2}} = \frac{q}{4\pi\varepsilon_0 R}\frac{1}{\sqrt{1+\left(\frac{z}{R}\right)^2}} \approx \frac{q}{4\pi\varepsilon_0 R}\left(1-\frac{z^2}{2R^2}\right).$$

A.3 (0.2 points)

The potential energy of the electron is $V(z) = -e\Phi(z)$. The force acting on the electron is

$$F(z) = -\frac{\mathrm{d}V(z)}{\mathrm{d}z} = +e\frac{\mathrm{d}\Phi}{\mathrm{d}z} = -\frac{qe}{4\pi\varepsilon_0 R^3}z.$$

If this is a restoring force, it should be negative for positive z. Thus, $q > 0$.

A.4 (0.1 points)

The equation of motion for an electron is

$$m\ddot{z} + \frac{qe}{4\pi\varepsilon_0 R^3}z = 0$$

(here dots denote time derivatives). We therefore get

$$\omega = \sqrt{\frac{qe}{4\pi m\varepsilon_0 R^3}}.$$

Part B. Electrostatic potential in the plane of the ring (1.7 points)

B.1 (1.5 points)

There are two different ways to solve this problem: (i) using direct integration; (ii) using Gauss's law and the result of part A.

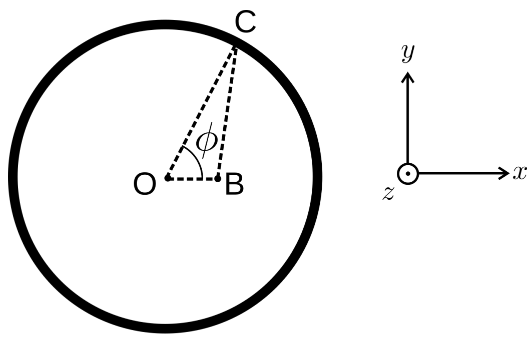


Figure 1: Calculating electrostatic potential in the plane of the ring through direct integration.

(i) **Direct integration**. We will follow the notations of Figure 1. Since the potential has cylindrical symmetry, let the point B, where we calculate the potential, be on the x-axis. Let

$$|\mathrm{OB}| = r; |\mathrm{OC}| = R.$$

Thus:

$$|\mathrm{BC}|^2 = R^2 + r^2 - 2Rr\cos\phi.$$

Electrostatic potential created by ring element $\mathrm{d}\phi$ at the point B:

$$\mathrm{d}\Phi = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R\,\mathrm{d}\phi}{\sqrt{R^2 + r^2 - 2Rr\cos\phi}} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\,\mathrm{d}\phi}{\sqrt{1 + \frac{r^2}{R^2} - 2\frac{r}{R}\cos\phi}}.$$

Using the expansion given in the formulation of the problem for $\varepsilon = -1/2$ we have:

$$\mathrm{d}\Phi \approx \frac{\lambda\,\mathrm{d}\phi}{4\pi\varepsilon_0}\left[1 - \frac{1}{2}\left(\frac{r^2}{R^2} - 2\frac{r}{R}\cos\phi\right) + \frac{3}{8}\left(\frac{r^2}{R^2} - 2\frac{r}{R}\cos\phi\right)^2\right].$$

Ignoring the terms of the order r^3 and r^4 we get:

$$\mathrm{d}\Phi \approx \frac{\lambda\,\mathrm{d}\phi}{4\pi\varepsilon_0}\left[1 + \frac{r}{R}\cos\phi + \frac{r^2}{R^2}\left(\frac{3}{2}\cos^2\phi - \frac{1}{2}\right)\right].$$

Integrating over all angles we finally obtain:

$$\Phi(r) = \frac{\lambda}{4\pi\varepsilon_0}\int_0^{2\pi}\left[1 + \frac{r}{R}\cos\phi + \frac{r^2}{R^2}\left(\frac{3}{2}\cos^2\phi - \frac{1}{2}\right)\right]\mathrm{d}\phi.$$

$$\Phi(r) = \frac{q}{4\pi\varepsilon_0 R}\left(1 + \frac{r^2}{4R^2}\right).$$

From here, comparing with the expression $\Phi(r) = q(\alpha + \beta r^2)$, we obtain

$$\beta = \frac{1}{16\pi\varepsilon_0 R^3}.$$

(ii) **Gauss's law.**

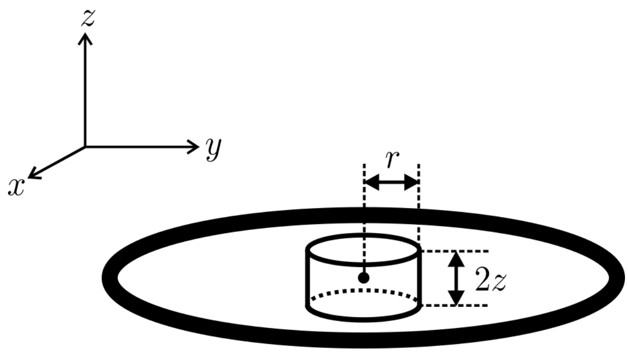


Figure 2: Calculating electrostatic potential in the plane of the ring via Gauss's law.

Let us analyze a small cylinder of radius r. The center of the cylinder coincides with the center of the ring. In part A we analyzed the potential along the z-axis, while in this part we analyze the potential along the radius r. For any $z \ll R$ and $r \ll R$ the potential has an expression:

$$\Phi(z, r) = \frac{q}{4\pi\varepsilon_0 R}\left(1 - \frac{z^2}{2R^2}\right) + q\beta r^2.$$

The lowest order terms are quadratic in r and z. Due to reflection symmetry the potential does not contain terms of the type rz. This, for example, immediately gives us $\alpha = 1/(4\pi\varepsilon_0 R)$. Thus, for small r and z electric fields in the radial and axial directions are:

$$\mathcal{E}_z(z, r) = +\frac{q}{4\pi\varepsilon_0 R^3} z, \qquad \mathcal{E}_r(z, r) = -2q\beta r.$$

Applying Gauss's law to the cylinder we obtain:

$$\oint \vec{\mathcal{E}} \cdot \mathrm{d}\vec{S} = 0 \qquad \Rightarrow \qquad \int\limits_{\text{side}} \vec{\mathcal{E}} \cdot \mathrm{d}\vec{S} + \int\limits_{\text{base}} \vec{\mathcal{E}} \cdot \mathrm{d}\vec{S} = 0.$$

The second integral is:

$$\int\limits_{\text{base}} \vec{\mathcal{E}} \cdot \mathrm{d}\vec{S} = 2\pi r^2 \mathcal{E}_z(z, r) = \frac{qzr^2}{2\varepsilon_0 R^3}.$$

The first integral is:

$$\int\limits_{\text{side}} \vec{\mathcal{E}} \cdot \mathrm{d}\vec{S} = 4\pi rz \mathcal{E}_r(z, r) = -8\pi q\beta r^2 z.$$

Gauss's theorem thus gives:

$$\frac{qzr^2}{2\varepsilon_0 R^3} - 8\pi q\beta r^2 z = 0.$$

This immediately yields

$$\beta = \frac{1}{16\pi\varepsilon_0 R^3},$$

which agrees with the result obtained via direct integration.

B.2 (0.2 points)

The potential of the electron is $V(r) = -e\Phi(r)$. Force acting on the electron in the xy plane is

$$F(r) = -\frac{\mathrm{d}V(r)}{\mathrm{d}r} = +e\frac{\mathrm{d}\Phi(r)}{\mathrm{d}r} = \frac{qe}{8\pi\varepsilon_0 R^3}r.$$

To have oscilations we need the force to be negative for $r > 0$. Thus, $q < 0$.

Part C. The focal length of the idealized electrostatic lens (2.3 points)

C.1 (1.3 points)

Let us consider an electron with the velocity $v = \sqrt{2E/m}$ at a distance r from the "optical" axis (Figure 2 of the problem). The electron crosses the "active region" of the lens in time

$$t = \frac{d}{v}.$$

The equation of motion in the r direction:

$$m\ddot{r} = 2eq\beta r.$$

During the time the electron crosses the active region of the lens, the electron acquires radial velocity:

$$v_r = \frac{2eq\beta r}{m}\frac{d}{v} < 0.$$

The lens will be focusing if $q < 0$. The time it takes for an electron to reach the "optical" axis is:

$$t' = \frac{r}{|v_r|} = -\frac{mv}{2eq\beta d}.$$

During this time the electron travels in the z-direction a distance

$$\Delta z = t'v = -\frac{mv^2}{2eq\beta d} = -\frac{E}{eqd\beta}.$$

Δz does not depend on the radial distance r, therefore all electron will cross the "optical" axis (will be focused) in the same spot. Thus,

$$f = -\frac{E}{eqd\beta}.$$

C.2 (0.8 points)

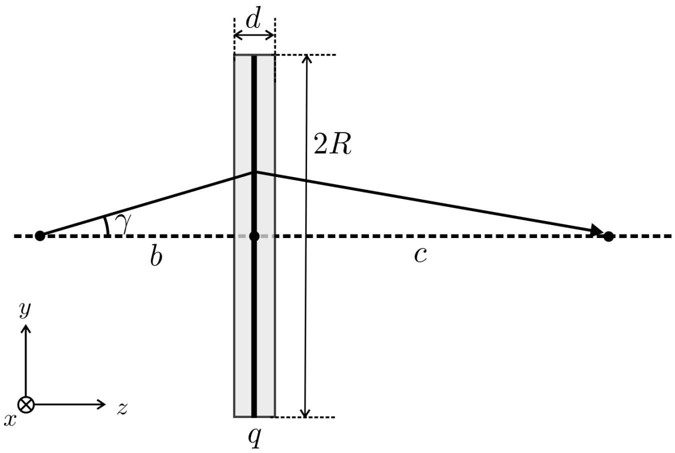


Figure 3: Focusing of electrons.

Let us consider an electron emitted an an angle γ to the optical axis (Figure 3). Its initial velocity in the radial direction is:

$$v_{r;0} = v\sin\gamma \approx v\gamma \approx v\frac{r}{b},$$

where r is the radial distance of the electron when it reaches the plane of the ring. The velocity in the z-direction is

$$v_z = v\cos\gamma \approx v.$$

For small angles γ the additional velocity in the r-direction acquired in the "active region" is the same as in part C.1. Thus, the radial velocity after crossing the active region is

$$v_r = v\frac{r}{b} + \frac{2eq\beta r}{m}\frac{d}{v},$$

where the first term is positive and the second term is negative, since $q < 0$. If the electrons are focused, then $v_r < 0$ (this can be verified after obtaining the final result). The electron will reach the optical axis in time

$$t' = \frac{r}{|v_r|} = -\frac{r}{\frac{2eq\beta r}{m}\frac{d}{v} + v\frac{r}{b}} = -\frac{1}{\frac{2eq\beta}{m}\frac{d}{v} + \frac{v}{b}}.$$

During this time it will travel a distance

$$c = t'v = -\frac{1}{\frac{2eq\beta}{m}\frac{d}{v^2} + \frac{1}{b}} = -\frac{1}{\frac{eq\beta d}{E} + \frac{1}{b}}.$$

C.3 (0.2 pt)

From the previous answer we obtain:

$$\frac{1}{b} + \frac{1}{c} = -\frac{eq\beta d}{E}.$$

Comparing with the answer of C.1 we immediately obtain

$$\frac{1}{b}+\frac{1}{c}=\frac{1}{f},$$

i.e. the equation of a thin optical lens is valid for an electrostatic lens as well.

Part D. The ring as a capacitor (3 points)

D.1 (2.0 points)

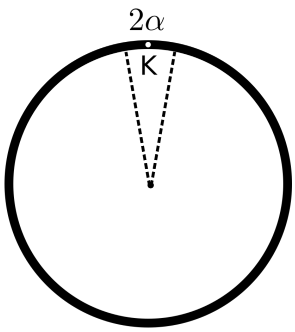


Figure 4: Calculation of the capacitance of the ring.

Let us sub-divide the entire ring into two parts: a part corresponding to the angle $2\alpha \ll 1$, and the rest of the ring, as shown in Figure 4. While the angle is small in comparison to 1, let us assume that the length of the first part, αR, is still large compared to a ($\alpha R \gg a$). Let us calculate the electrostatic potential Φ at point K. It it a sum of two terms: the first one produced by the cut-out part with an angle 2α (contribution Φ_1) and the second one originating from the rest of the ring (contribution Φ_2).

Contribution Φ_1. Since $\alpha \ll 1$, we can neglect the curvature of the cylinder that is cut out from the ring. The linear charge density on the ring is $\lambda = \frac{q}{2\pi R}$. The potential at the center of the cylinder is then given by an integral:

$$\Phi_1 = 2\frac{1}{4\pi\varepsilon_0}\frac{q}{2\pi R}\int_0^{\alpha R}\frac{\mathrm{d}x}{\sqrt{x^2+a^2}} = \frac{q}{4\pi^2\varepsilon_0 R}\int_0^{\alpha R}\frac{\mathrm{d}(x/a)}{\sqrt{1+(x/a)^2}} = \frac{q}{4\pi^2\varepsilon_0 R}\int_0^{\alpha R/a}\frac{\mathrm{d}y}{\sqrt{1+y^2}}.$$

Using the integral provided in the description of the problem we get:

$$\Phi_1 = \frac{q}{4\pi^2\varepsilon_0 R}\ln\left(y+\sqrt{1+y^2}\right)\Big|_0^{\alpha R/a} = \frac{q}{4\pi^2\varepsilon_0 R}\ln\left(\frac{\alpha R}{a}+\sqrt{1+\left(\frac{\alpha R}{a}\right)^2}\right).$$

As $\alpha R \gg a$,

$$\Phi_1 \approx \frac{q}{4\pi^2\varepsilon_0 R}\ln\left(\frac{2\alpha R}{a}\right).$$

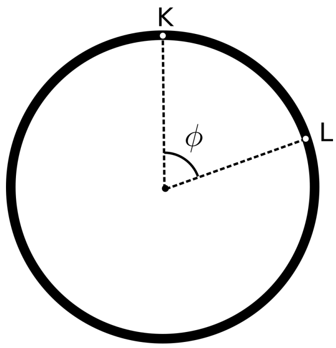


Figure 5: Calculation of the capacitance of the ring

Contribution Φ_2. In this case we can neglect the thickness a. Using the cosine theorem we can derive the distance between points K and L of Figure 5:

$$|\mathrm{KL}| = 2R\sin\frac{\phi}{2}.$$

The contribution Φ_2 can then be written as an integral:

$$\Phi_2 = 2\frac{q}{2\pi}\frac{1}{4\pi\varepsilon_0}\int_\alpha^\pi \frac{\mathrm{d}\phi}{2R\sin\frac{\phi}{2}} = \frac{q}{8\pi^2\varepsilon_0 R}\int_\alpha^\pi \frac{\mathrm{d}\phi}{\sin\frac{\phi}{2}} = \frac{q}{4\pi^2\varepsilon_0 R}\int_\alpha^\pi \frac{\mathrm{d}\left(\frac{\phi}{2}\right)}{\sin\frac{\phi}{2}} = \frac{q}{4\pi^2\varepsilon_0 R}\int_{\alpha/2}^{\pi/2}\frac{\mathrm{d}\chi}{\sin\chi}.$$

Using the integral from the formulation of the problem, we calculate:

$$\int_{\alpha/2}^{\pi/2}\frac{\mathrm{d}\chi}{\sin\chi} = -\ln\left(\frac{\cos\chi+1}{\sin\chi}\right)\Bigg|_{\alpha/2}^{\pi/2} = \ln\left(\frac{\cos\alpha/2+1}{\sin\alpha/2}\right) \approx \ln\left(\frac{4}{\alpha}\right)$$

for $\alpha \ll 1$. Therefore

$$\Phi_2 \approx \frac{q}{4\pi^2\varepsilon_0 R}\ln\left(\frac{4}{\alpha}\right).$$

The total potential and capacitance. The total potential is the sum of Φ_1 and Φ_2:

$$\Phi = \Phi_1 + \Phi_2 = \frac{q}{4\pi^2\varepsilon_0 R}\ln\left(\frac{2\alpha R}{a}\right) + \frac{q}{4\pi^2\varepsilon_0 R}\ln\left(\frac{4}{\alpha}\right) = \frac{q}{4\pi^2\varepsilon_0 R}\ln\left(\frac{8R}{a}\right).$$

α drops out from the expression. From here we obtain the capacitance $C = q/\Phi$:

$$C = \frac{4\pi^2\varepsilon_0 R}{\ln\left(\frac{8R}{a}\right)}.$$

$C \to 0$ as $a \to 0$.

D.2 (1.0 point)

Let $q(t)$ be the charge on the ring at a time t. Potential of the disk is thus $q(t)/C$. Voltage drop of the resistor is $R_0 I(t) = R_0\,\mathrm{d}q/\mathrm{d}t$. Therefore for time $-\frac{d}{2v} < t < \frac{d}{2v}$:

$$\frac{q(t)}{C} + R_0\frac{\mathrm{d}q}{\mathrm{d}t} = V_0.$$

Integrating this equation and keeping in mind that $q(t) = 0$ at $t = -d/(2v)$, we get:

$$q(t) = CV_0\left(1 - \mathrm{e}^{-\frac{d}{2vR_0C}}\mathrm{e}^{-\frac{t}{R_0C}}\right).$$

The charge attains the largest absolute value at $t = d/(2v)$. The value of the charge at this time is:

$$q_0 = CV_0\left(1 - \mathrm{e}^{-\frac{d}{vR_0C}}\right).$$

When $t > \frac{d}{2v}$, we get:

$$\frac{q(t)}{C} + R_0\frac{\mathrm{d}q}{\mathrm{d}t} = 0.$$

From here:

$$q(t) = q_0\mathrm{e}^{-\frac{t}{R_0C}+\frac{d}{2vR_0C}} = CV_0\left(\mathrm{e}^{\frac{d}{2vR_0C}} - \mathrm{e}^{-\frac{d}{2vR_0C}}\right)\mathrm{e}^{-\frac{t}{RC}}.$$

Therefore, we obtain:

$$q(t) = \begin{cases} 0 & \text{for } t < -\frac{d}{2v}; \\ CV_0\left(1 - \mathrm{e}^{-\frac{d}{2vR_0C}}\mathrm{e}^{-\frac{t}{R_0C}}\right) & \text{for } -\frac{d}{2v} < t < \frac{d}{2v}; \\ CV_0\left(\mathrm{e}^{\frac{d}{2vR_0C}} - \mathrm{e}^{-\frac{d}{2vR_0C}}\right)\mathrm{e}^{-\frac{t}{R_0C}} & \text{for } t > \frac{d}{2v}. \end{cases}$$

For a lens to be focusing we require that charge is negative, therefore $V_0 < 0$. The dependence of charge on time is shown in Figure 6.

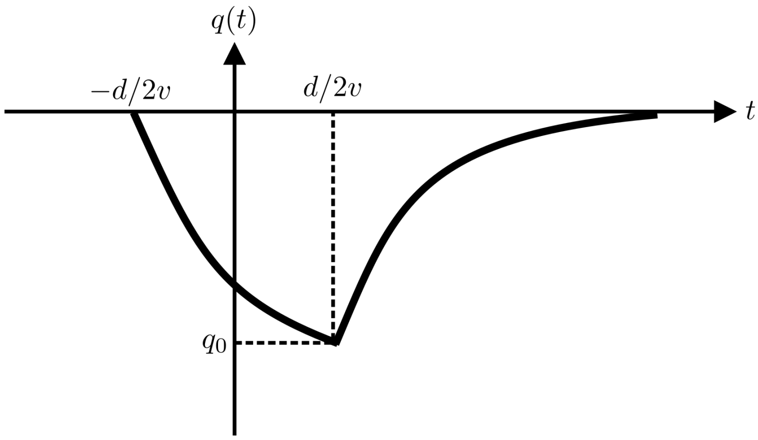


Figure 6: Charge on the ring as a function of time.

Part E. Focal length of a more realistic lens (2 points)

E.1 (1.7 points)

Like in part C, the radial equation of motion of an electron is:

$$m\ddot{r} = 2eq(t)\beta r,$$

where in this case $q(t)$ depends on time. Using the notation $\eta = 2e\beta/m$, we obtain:

$$\ddot{r} - \eta q(t) r = 0.$$

As $f/v \gg R_0C$, then during charging–decharging the electron does not substantially change its radial position r, and we can assume r to be constant during the entire charging–decharging process. In this case the acquired vertical velocity is

$$v_r = \eta r \int_{-d/(2v)}^{\infty} q(t)\,\mathrm{d}t.$$

We can use the derived equations for $q(t)$ and find the integrals. The integral $\int_{-d/(2v)}^{d/(2v)} q(t)\,\mathrm{d}t$ is (using the notation $d/v = t_0$, $R_0C = \tau$, $CV_0 = Q_0$):

$$\int_{-t_0/2}^{t_0/2} q(t)\,\mathrm{d}t = \int_{-t_0/2}^{t_0/2} Q_0\left(1 - \mathrm{e}^{-\frac{t_0}{2\tau}}\mathrm{e}^{-\frac{t}{\tau}}\right)\mathrm{d}t = Q_0\left(t_0 - \tau\left[1 - \mathrm{e}^{-t_0/\tau}\right]\right).$$

The integral $\int_{d/(2v)}^{\infty} q(t)\,\mathrm{d}t$ is

$$\int_{t_0/2}^{\infty} Q_0\left(\mathrm{e}^{\frac{t_0}{2\tau}} - \mathrm{e}^{-\frac{t_0}{2\tau}}\right)\mathrm{e}^{-\frac{t}{\tau}}\,\mathrm{d}t = Q_0\tau\left[1 - \mathrm{e}^{-t_0/\tau}\right].$$

Adding the two integrals we obtain for the final integral:

$$\int_{-t_0/2}^{\infty} q(t)dt = Q_0 t_0.$$

Interestingly, it does not depend on $\tau = R_0C$. Therefore, the acquired vertical velocity of the electron is

$$v_r = \eta r\frac{CV_0 d}{v} = \frac{2e\beta CV_0 dr}{mv}.$$

Following the logic similar to part C, we derive the focal length

$$f = -\frac{E}{eCV_0 d\beta}.$$

E.2 (0.3 points).

Comparing $f = -E/(eCV_0 d\beta)$ with $f = -E/(eqd\beta)$ from part C we immediataly obtain $q_{\text{eff}} = CV_0$.

Particles and Waves (10 points)

Wave–particle duality, which states that each particle can be described as a wave and vice versa, is one of the central concepts of quantum mechanics. In this problem, we will rely on this notion and just a few other basic assumptions to explore a selection of quantum phenomena covering the two distinct types of particles of the microworld—fermions and bosons.

Part A. Quantum particle in a box (1.4 points)

Consider a particle of mass m moving in a one-dimensional potential well, where its potential energy $V(x)$ is given by

$$V(x) = \begin{cases} 0, & 0 \leq x \leq L; \\ \infty, & x < 0 \text{ or } x > L. \end{cases} \quad (1)$$

While classical particle can move in such a potential having any kinetic energy, for quantum particle only some specific positive discrete energy levels are allowed. In any such allowed state, the particle can be described as a standing de Broglie wave with nodes at the walls.

A.1	Determine the minimal possible energy E_{min} of the quantum particle in the well. Express your answer in terms of m, L, and the Planck's constant h.	0.4pt

The particle's state with minimal possible energy is called the ground state, and all the rest allowed states are called excited states. Let us sort all the possible energy values in the increasing order and denote them as E_n, starting from E_1 for the ground state.

A.2	Find the general expression for the energy E_n (here $n = 1, 2, 3, \ldots$).	0.6pt

A.3	Particle can undergo instantaneous transition from one state to another only by emitting or absorbing a photon of the corresponding energy difference. Find the wavelength λ_{21} of the photon emitted during the transition of the particle from the first excited state (E_2) to the ground state (E_1).	0.4pt

Part B. Optical properties of molecules (2.1 points)

In this part, we will study several optical properties of the cyanine Cy5 molecule—a widely used dye molecule, schematically shown in Fig. 1a. Its optical properties are determined mainly by the carbon backbone, composed of the alternating single and double bonds between carbon atoms, shown in Fig. 1b, while the influence of the rings at the molecule's ends as well as radicals R is much smaller. Three of the four valence electrons of each C atom (and of N atoms) in the backbone form the chemical bonds, while the remaining valence electrons are "shared" and can move along the whole backbone. The net potential energy of each such electron is shown with oscillating thin line in Fig. 1c, with minima corresponding to the positions of the C and N atoms.

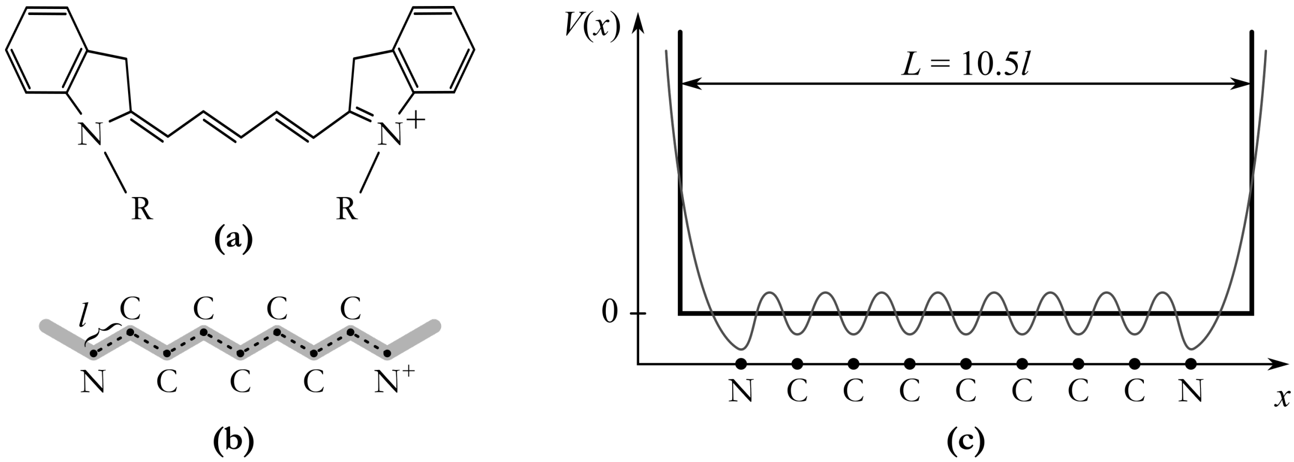


Figure 1. (a) Chemical structure of the cyanine Cy5 molecule (for simplicity, hydrogen atoms are not shown, and R denote some radicals). (b) The backbone of the Cy5 molecule, with mean inter-atomic distance l . (c) Potential energy of the electron along the backbone (thin line) and its approximation by the step function given by Eq. 1 (thick line).

For simplicity, we will approximate this potential energy profile by a simple function given in Eq. 1 with the width $L = 10.5l$ (see thick line in Fig. 1c), here $l = 140\ \text{pm}$ is the mean inter-atomic distance (see also Fig. 1b). As a result, we obtain the "electronic gas" composed of 10 electrons (7 from C atoms, 2 from the N atom, and 1 from the N^+ ion), moving in a one-dimensional potential well discussed in part A. In our evaluation, we can neglect the mutual interaction of these electrons; however, we should account for the fact that electrons are fermions and thus obey the Pauli exclusion principle. We also neglect the influence of other electrons as well as motion of the nuclei.

B.1	Evaluate the largest wavelength λ of the photon that can be absorbed by the Cy5 molecule assuming that the electron system is initially in its ground state. Express your answer in terms of l , physical constants and some numerical prefactor, and calculate the numerical value.	0.8pt

B.2	Another dye molecule Cy3 has similar structure, but its backbone is shorter by 2 carbon atoms. Is its absorption spectrum shifted to the bluer or to the redder spectral region compared to the Cy5 molecule? Evaluate numerically the magnitude $\Delta\lambda$ of this spectral shift. You can assume that removing two carbon atoms doesn't change the molecule shape and only makes the backbone length shorter by two interatomic distances.	0.4pt

Being in the excited state, molecule can undergo a spontaneous transition to the ground state while emitting photon. The mean rate K of such events (i.e. the relative decrease of the molecules being in the excited state, $\mathrm{d}N/N$, over time $\mathrm{d}t$, $K = \frac{1}{N}\frac{\mathrm{d}N}{\mathrm{d}t}$) is determined by the wavelength λ of the emitted photon, the transition electrical dipole moment d (which is of the order of $d \simeq el$, here e is elementary charge) as well as vacuum permittivity ε_0 and Planck's constant h.

B.3	Using dimension analysis, determine the expression for the rate of spontaneous emission in terms of ε_0, h, λ, and d. The numerical prefactor for your expression is $= \frac{16}{3}\pi^3$.	0.7pt

B.4	For Cy5 molecule, $d \approx 2.4\ el$. Evaluate the mean fluorescence lifetime of the lowest excited state of Cy5 molecule, τ_{Cy5}, which is reciprocal to the rate of its emissive transition to the ground state.	0.2pt

Part C. Bose-Einstein condensation (1.5 points)

This part is not directly related to Parts A and B. Here, we will study the collective behaviour of bosonic particles. Bosons do not respect the Pauli exclusion principle, and—at low temperatures or high densities—experience a dramatic phenomenon known as the Bose–Einstein condensation (BEC). This is a phase transition to an intriguing collective quantum state: a large number of identical particles 'condense' into a single quantum state and start behaving as a single wave. The transition is typically reached by cooling a fixed number of particles below the critical temperature. In principle, it can also be induced by keeping the temperature fixed and driving the particle density past its critical value.

We begin by exploring the relation between the temperature and the particle density at the transition. As it turns out, estimates of their critical values can be deduced from a simple observation: *Bose-Einstein condensation takes place when the de Broglie wavelength corresponding to the mean square speed of the particles is equal to the characteristic distance between the particles in a gas*.

C.1	Given a non-interacting gas of ^{87}Rb atoms in thermal equilibrium, write the expressions for their typical linear momentum p and the typical de Broglie wavelength λ_{dB} as a function of atom's mass m, temperature T and physical constants.	0.4pt

C.2	Calculate the typical distance between the particles in a gas, ℓ, as a function of particle density n. Hence deduce the critical temperature T_c as a function of atom's mass, their density and physical constants.	0.5pt

To realize BEC in the lab, the experimentalists have to cool gases to temperatures as low as T_c = 100 nK.

C.3	What is the particle density of the Rb gas n_c if the transition takes place at such a temperature? For the sake of comparison, calculate also the 'ordinary' particle density n_0 of an ideal gas at the standard temperature and pressure (STP), i.e. $T_0 = 300\ \text{K}$ and $p_0 = 10^5\ \text{Pa}$. How many times is the 'ordinary' gas denser? You may assume that the mass of the atoms is equal to 87 atomic mass units (m_{amu}).	0.6pt

Part D. Three-beam optical lattices (5 points)

The first Bose-Einstein condensates were produced back in 1995, and since then the experimental work has branched out in diverse directions. In this part, you will investigate one particularly fruitful idea to load the condensate into spatially periodic potentials created by interfering a number of coherent laser beams. Due to the periodic nature of the resulting interference patterns, they are referred to as *optical lattices*. The potential energy $V(\vec{r})$ of an atom moving in an optical lattice is proportional to the local intensity of the light, and in your calculations you may assume that

$$V(\vec{r}) = -\alpha \left\langle |\vec{E}(\vec{r},t)|^2 \right\rangle. \tag{2}$$

Here, α is a *positive* constant, and the angle brackets indicate time-averaging which eliminates the rapidly oscillating terms. The electric field produced by the *i*-th laser is described by

$$\vec{E}_i = E_{0,i}\,\vec{\varepsilon}_i \cos(\vec{k}_i \cdot \vec{r} - \omega t), \tag{3}$$

with the amplitude $E_{0,i}$, the wave vector $\vec{k}_i$, and the unit-length polarization vector $\vec{\varepsilon}_i$.

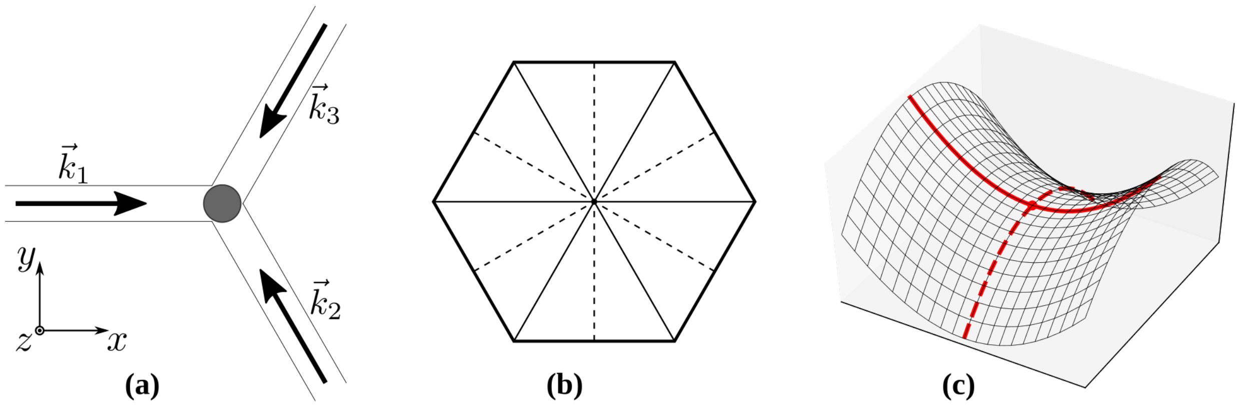


Figure 2. (a) Three-beam optical lattice: three plane waves with wave vectors $\vec{k}_{1,2,3}$ intersect and interfere in the area indicated by the grey circle. (b) Symmetries of a regular hexagon: solid and dashed lines show two sets of symmetry axes. (c) Saddle point: a point on a surface where the slopes in orthogonal directions are all zero, but which is not a local extremum of the plotted function. Travelling along the trajectory marked by the full line one encounters an apparent minimum. Additional analysis of the perpendicular direction (dashed line) is needed to distinguish a true minimum from a saddle point (shown).

Your task is to study *triangular optical lattices* that are produced by interfering three coherent laser beams of equal intensity. A typical setup is shown in Fig. 2a. Here, all three beams are polarized in the z direction, propagate in the xy plane and intersect at equal angles of 120°. Choose the direction of the x axis parallel to the wave vector $\vec{k}_1$.

D.1	Using Eqs. 2 and 3 obtain the expression for the potential energy $V(\vec{r})$ as a function of $\vec{r} = (x, y)$ in the plane of the beams. *Hint*: the result can be neatly expressed as a constant term plus a sum of three cosine functions of arguments $\vec{b}_i \cdot \vec{r}$. Please write your result in this form and identify the vectors $\vec{b}_i$.	1.4pt

D.2	The resulting potential energy has a sixfold rotational symmetry axis, i.e., the potential distribution is invariant with respect to a rotation by a multiple of 60° around the origin. Provide a simple argument to prove that this is indeed the case.	0.5pt

The above observation of symmetry simplifies the analysis of the two-dimensional potential distribution $V(\vec{r})$. As shown in Fig. 2b, a regular hexagon has symmetry lines that, respectively, connect opposite vertices (solid lines) and midpoints of opposite edges (dashed lines). Therefore, in our situation one does not need to produce and study two-dimensional potential plots as many insights can be deduced by focusing on the coordinate axes x and y that run along the symmetry lines.

D.3	Derive the behavior of the potential $V(\vec{r})$ along the coordinate axes, i.e., determine the functions $V_X(x) \equiv V(x,0)$ and $V_Y(y) \equiv V(0,y)$. Identify the locations of the extrema of $V_X(x)$ and $V_Y(y)$ as functions of a single argument. As these functions are periodic, please include in your lists only one representative from each family of periodically repeated minima and maxima.	1.2pt

We are interested in determining the locations of so-called *lattice sites*, i.e., the minima of the full two-dimensional potential $V(\vec{r})$. The obtained minima of single-argument functions V_X and V_Y identify their suspected positions but still have to be checked to eliminate the saddle points. As shown in Fig. 2c, when studied along a single line, saddle points may disguise as minima but they are not.

D.4	Review your results in the previous question to determine actual minima of the optical lattice: Identify all equivalent minima nearest to (but not coinciding with) the origin. What is the distance a between the nearest minima, in other words—the *lattice constant* of our optical lattice? Express the answer in terms of the laser wavelength λ_{las}.	0.8pt

Charge neutrality of ultracold atoms suggests that their interactions become relevant only when two or more atoms occupy the same site of an optical lattice. However, experimentalists are also able to explore setups that sustain long-range atomic interactions. A possible approach relies on creation of the so-called *Rydberg atoms* that are physically large and feature other exaggerated properties. Rydberg atoms are excited atoms with one electron promoted to a state with a very high principal quantum number n. The size of a Rydberg atom can be estimated by calculating the radius of the classical circular orbit of that electron with the orbital angular momentum $n\hbar$, here $\hbar$ is the reduced Planck constant.

D.5	Calculate the value of n that corresponds to the radius of the Rb Rydberg atom comparable to the wavelength of laser light λ_{las} = 380 nm. Give your answer in terms of λ_{las} and physical constants and find its numerical value.	1.1pt

Particles and Waves (10 points)

Part A. Quantum particle in a box (1.4 points)

A.1 (0.4 pt)

$E_{\text{min}} =$

A.2 (0.6 pt)

$E_n =$

A.3 (0.4 pt)

$\lambda_{21} =$

Part B. Optical properties of molecules (2.1 points)

B.1 (0.8 pt)

Expression: $\lambda =$

Numerical value: $\lambda \approx$

B.2 (0.4 pt)
Absorption spectrum of Cy3 is shifted to (check): ☐ bluer ☐ redder
spectral region by $\Delta\lambda \approx$

B.3 (0.7 pt)

$K =$

B.4 (0.2 pt)

Numerical value: $\tau_{\text{Cy5}} \approx$

Part C. Bose-Einstein condensation (1.5 points)

C.1 (0.4 pt)

$p =$

$\lambda_{\mathrm{dB}} =$

C.2 (0.5 pt)

$\ell =$

$T_c =$

C.3 (0.6 pt)
Expression: $n_c =$

Numerical value: $n_c \approx$

Expression: $n_0 =$

Numerical value: $n_0/n_c \approx$

Part D. Three-beam optical lattices (5 points)

D.1 (1.4 pt)

$V(\vec{r}) =$

$\vec{b}_1 =$

$\vec{b}_2 =$

$\vec{b}_3 =$

D.2 (0.5 pt)
Argument:

D.3 (1.2 pt)

$V_{\mathrm{X}}(x) =$

$V_{\mathrm{Y}}(y) =$

Minimum (-a) of $V_{\mathrm{X}}(x)$: at $x =$

Maximum (-a) of $V_{\mathrm{X}}(x)$: at $x =$

Minimum (-a) of $V_{\mathrm{Y}}(y)$: at $y =$

Maximum (-a) of $V_{\mathrm{Y}}(y)$: at $y =$

D.4 (0.8 pt)
Ratio of the lattice constant to the laser wavelength: $a/\lambda_{\mathrm{las}} =$

Positions of all equivalent minima nearest to the origin:

D.5 (1.1 pt)

Expression: $n =$

Numerical value: $n \approx$

Particles and Waves (10 points)

Part A. Quantum particle in a box (1.4 points)

A.1 (0.4 points)

The width of the potential well (L) should be equal to the half of the wavelength of the de Broglie standing wave $\lambda_{\text{dB}} = h/p$, here h is the Planck's constant and p is the momentum of the particle. Thus $p = h/\lambda_{\text{dB}} = h/(2L)$, and the minimal possible energy of the particle is

$$E_{\min} = \frac{p^2}{2m} = \frac{h^2}{8mL^2}.$$

A.2 (0.6 points)

The potential well should fit an integer number of the de Broglie half-wavelengths: $L = \frac{1}{2}\lambda_{\text{dB}}^{(n)} \cdot n$, $n = 1, 2, \ldots$. Therefore, particle's momentum, corresponding to the de Broglie wavelength $\lambda_{\text{dB}}^{(n)}$ is

$$p_n = \frac{h}{\lambda_{\text{dB}}^{(n)}} = \frac{hn}{2L},$$

and the corresponding energy is

$$E_n = \frac{p_n^2}{2m} = \frac{h^2 n^2}{8mL^2}, \qquad n = 1, 2, 3, \ldots. \tag{1}$$

A.3 (0.4 points)

The energy of the emitted photon, $E = hc/\lambda$ (here c is the speed of light and λ is the photon's wavelength) should be equal to the energy difference $\Delta E = E_2 - E_1$, therefore

$$\lambda_{21} = \frac{hc}{E_2 - E_1} = \frac{8mcL^2}{3h}.$$

Part B. Optical properties of molecules (2.1 points)

B.1 (0.8 points)

Taking into account the Pauli exclusion principle, each energy level E_n is occupied by two electrons with spins oriented in the opposite directions. As a results, 10 electrons fill the lowest 5 states, and the absorption of the photon of the longest wavelength corresponds to the transition of one electron from the occupied E_5 to the unoccupied E_6 energy state:

$$\frac{hc}{\lambda} = E_6 - E_5,$$

where E_6 and E_5 can be found from Eq. 1, where m is replaced with the electron mass m_e. Hence we obtain:

$$\lambda = \frac{c \cdot 8 m_e L^2}{h\,(6^2 - 5^2)} = \frac{10.5^2 \cdot 8}{11} \frac{m_e c l^2}{h} = \frac{882}{11} \frac{m_e c l^2}{h} \approx 647\,\text{nm}.$$

This result correspond precisely to the experimental value the peak position of the Cy5 absorption spectrum.

B.2 (0.4 points)

In the similar model for the Cy3 molecule, there are 8 electrons in the box of length $L = 8.5l$, thus photon's absorption corresponds to the $E_4 \longrightarrow E_5$ transition. Taking into account the result of question B1, we obtain

$$\lambda_{\text{Cy3}} = \frac{8.5^2 \cdot 8}{(5^2 - 4^2)} \frac{m_e c l^2}{h} \approx 518\,\text{nm},$$

i. e. the absorption spectrum of the Cy3 molecule is shifted by $\Delta\lambda \approx 129\,\text{nm}$ to the blue comparing to that of the Cy5 molecule. The experimental value is $\lambda_{\text{Cy3}}^{(\text{exp})} = 548\,\text{nm}$, so that our model catches general properties of these dye molecules rather well.

B.3 (0.7 points)

Let us assume

$$K = k \varepsilon_0^{\alpha} h^{\beta} \lambda^{\gamma} d^{\delta}. \tag{2}$$

The SI units of the relevant quantities are:

$$[\varepsilon_0] = \frac{\text{A}^2 \cdot \text{s}^4}{\text{kg} \cdot \text{m}^3}, \qquad [h] = \frac{\text{kg} \cdot \text{m}^2}{\text{s}}, \qquad [\lambda] = \text{m}, \qquad [d] = \text{A} \cdot \text{s} \cdot \text{m}, \qquad [K] = \text{s}^{-1}.$$

By plugging these expressions into Eq. 2 we obtain a simple system of linear equations for the unknown powers α, β, γ, and δ:

$$2\alpha + \delta = 0, \qquad -\alpha + \beta = 0, \qquad 4\alpha - \beta + \delta = -1, \qquad -3\alpha + 2\beta + \gamma + \delta = 0.$$

By solving this system we get:

$$\alpha = \beta = -1, \qquad \gamma = -3, \qquad \delta = 2,$$

so that the rate of spontaneous emission is

$$K = \frac{16\pi^3}{3} \frac{d^2}{\varepsilon_0 h \lambda^3}. \tag{3}$$

B.4 (0.2 points)

By using the result of question B.2 and expressing transition dipole moment as $d = 2.4\,el$, we obtain from Eq. 3:

$$\tau_{\text{Cy5}} = \frac{3}{16\pi^3} \frac{\varepsilon_0 h}{2.4^2 l^2 e^2} \lambda^3 \approx 3.3\,\text{ns}.$$

Part C. Bose-Einstein condensation (1.5 points)

C.1 (0.4 points)

At temperature T, the average kinetic energy of translational motion is $\frac{3}{2}k_B T$. Equating this result to $p^2/(2m)$, we obtain typical momentum $p = \sqrt{3mk_B T}$ and the de Broglie wavelength

$$\lambda_{dB} = \frac{h}{p} = \frac{h}{\sqrt{3mk_B T}}.$$

C.2 (0.5 points)

The volume per particle V/N is a good estimate for ℓ^3. We obtain $\ell = n^{-1/3}$, with $n = N/V$ and equate $\ell = \lambda_{dB}$ to express $T_c = h^2 n^{2/3}/(3mk_B)$.

C.3 (0.6 points)

Using the answer to the previous question, we express $n_c = (3mk_B T_c)^{3/2}/h^3$. Equation of state for the ideal gas gives $n_0 = p/(k_B T)$. Numerical estimations yield $n_c \approx 1.59 \cdot 10^{18}\ \mathrm{m}^{-3}$ and $n_0/n_c \approx 1.5 \cdot 10^7$.

Part D. Three-beam optical lattices (5.0 points)

D.1 (1.4 points)

We sum the three electric fields (z components)

$$E(\vec{r}, t) = E_0 \sum_{i=1}^{3} \cos\left(\vec{k}_i \cdot \vec{r} - \omega t\right), \tag{4}$$

and square the result

$$\begin{aligned} E^2(\vec{r}, t) &= E_0^2 \sum_{i=1}^{3} \sum_{j=1}^{3} \cos\left(\vec{k}_i \cdot \vec{r} - \omega t\right) \cos\left(\vec{k}_j \cdot \vec{r} - \omega t\right) \\ &= \frac{E_0^2}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} \left\{\cos\left[\left(\vec{k}_i - \vec{k}_j\right) \cdot \vec{r}\right] + \cos\left[\left(\vec{k}_i + \vec{k}_j\right) \cdot \vec{r} - 2\omega t\right]\right\}. \end{aligned} \tag{5}$$

Time averaging gives

$$\langle E^2(\vec{r}, t)\rangle = \frac{E_0^2}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} \cos\left[\left(\vec{k}_i - \vec{k}_j\right) \cdot \vec{r}\right], \tag{6}$$

we analyse the 9 terms and simplify to

$$\langle E^2(\vec{r},t)\rangle = E_0^2\left(\frac{3}{2}+\sum_{j=1}^{3}\cos\left(\vec{b}_j\cdot\vec{r}\right)\right). \tag{7}$$

Here

$$\vec{b}_1=\vec{k}_2-\vec{k}_3,\qquad \vec{b}_2=\vec{k}_3-\vec{k}_1,\qquad \vec{b}_3=\vec{k}_1-\vec{k}_2,$$

or in terms of the Levi-Civita symbol, $\vec{b}_k=\varepsilon_{ijk}(\vec{k}_i-\vec{k}_j)$. Incidentally, they are known as the reciprocal lattice vectors.

D.2 (0.5 points)

Argument: Observe that rotation by 60° maps the three vectors $\vec{b}_{1,2,3}$ into the relabelled triplet of $-\vec{b}$'s.

D.3 (1.2 points)

We find

$$V(x,y)=-\alpha E_0^2\left\{\frac{3}{2}+\cos\left(ky\sqrt{3}\right)+\cos\left(\frac{3kx}{2}+\frac{ky\sqrt{3}}{2}\right)+\cos\left(\frac{3kx}{2}-\frac{ky\sqrt{3}}{2}\right)\right\}, \tag{8}$$

and deduce

$$V_X(x)=-\alpha E_0^2\left\{\frac{5}{2}+2\cos\frac{3kx}{2}\right\}. \tag{9}$$

The potential has a simple cosine form, and the origin in an obvious minimum. Its replica appear at multiples of $\Delta x=4\pi/(3k)$. In the midpoint between any two minima, e.g. at $x=\Delta x/2=2\pi/(3k)$, the function $V_X(x)$ has its maxima.

Concerning the behaviour along the y axis, we have

$$V_Y(y)=-\alpha E_0^2\left\{\frac{3}{2}+\cos 2\varphi+2\cos\varphi\right\},\qquad \varphi=\sqrt{3}ky/2. \tag{10}$$

Looking for the extrema, we find the equation

$$\sin 2\varphi+\sin\varphi=0. \tag{11}$$

- $\varphi=0$ (corresponding to $y=0$) is the 'deep' minimum – the lattice site;
- $\varphi=\pi$ (corresponding to $y=\frac{2\pi}{\sqrt{3}k}$) is the 'shallow' minimum (later shown to be a saddle point of $V(x,y)$);
- $\varphi=2\pi/3$ and $\varphi=4\pi/3$ (corresponding to $y=\frac{4\pi}{3\sqrt{3}k}$ and $y=\frac{8\pi}{3\sqrt{3}k}$, respectively) are maxima.

D.4 (0.8 points)

We review the minima found in the previous question and eliminate the saddle point at $(0, 2\pi/3\sqrt{3}k)$. The actual minima of the 2D potential landscape $V(x, y)$ are:

- $(0, 0)$ – at the origin;
- $(4\pi/(3k), 0)$ – nearest to the origin in the positive direction along the x axis. On the grounds of symmetry we argue that there are six equivalent nearest minima in the directions $0°$, $\pm 60°$, $\pm 120°$, and $180°$ with respect to the x axis.

Distance between nearest minima (the lattice constant) $a = 4\pi/(3k)$. Given that the laser wavelength is $\lambda_{\text{las}} = 2\pi/k$, we have $a = \Delta x = 2\lambda_{\text{las}}/3$, thus $a/\lambda_{\text{las}} = 2/3$.

D.5 (1.1 points)

The atom's core electrons (all but the one promoted to to a state with a high principal quantum number n) shield the electric field of the nucleus so that the effective potential for the outer electron resembles that of a hydrogen atom. The attractive force acting on that electron, $F = e^2/(4\pi\varepsilon_0 r^2)$, gives rise to its centripetal acceleration $a = v^2/r$. Equating $F = m_e a$ and using the expression for the angular momentum $m_e v r = n\hbar$ to eliminate the velocity, we find the quantum number n corresponding to the orbit with the radius $r = \lambda_{\text{las}}$:

$$n = \frac{e}{\hbar}\sqrt{\frac{m_e \lambda}{4\pi\varepsilon_0}} \approx 85. \tag{12}$$