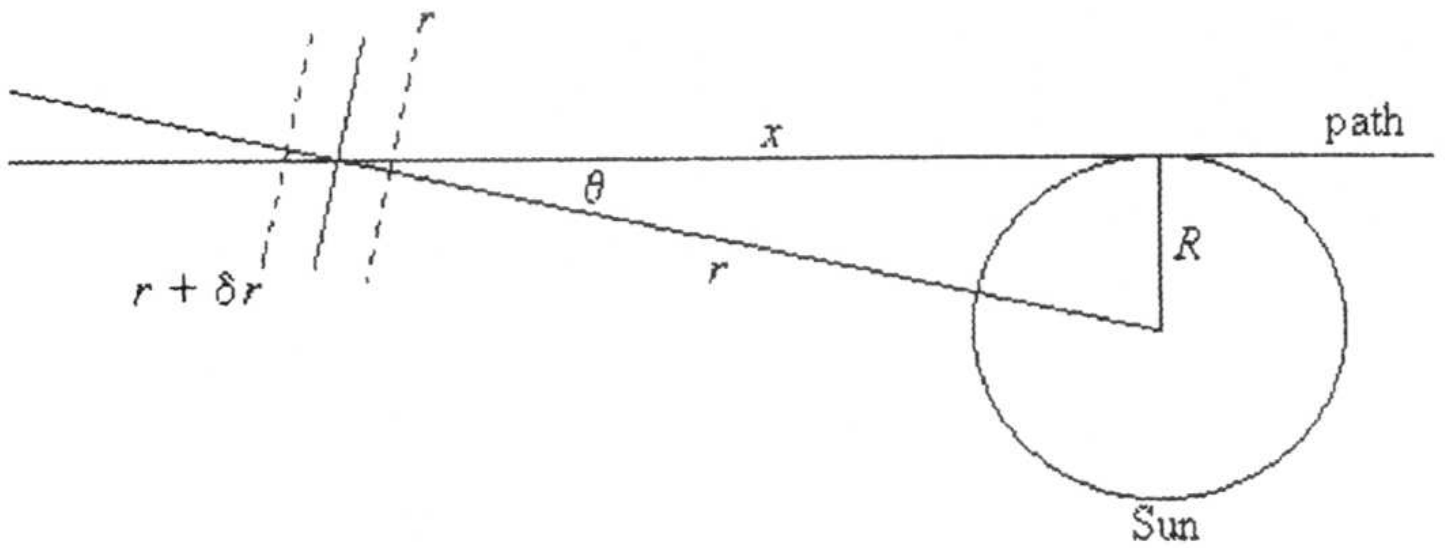


- A**
- a) $\Delta x_t = ae^{-\mu t} \cos(\omega t + \phi)$, $0.8 = e^{-50\mu} \Rightarrow \mu = 4.5 \times 10^{-3} \text{ s}^{-1}$.
- b) $v = (E/\rho)^{1/2} = (7.1 \times 10^{10}/2700)^{1/2} = 5100 \text{ m.s}^{-1}$.
 At fundamental $\lambda_{rod} = 4l = 4 \text{ m}$.
 $f = 5100 / 4 = 1.3 \times 10^3 \text{ Hz}$.
 $\omega = 2\pi f = 8.1 \times 10^3 \text{ rad.s}^{-1}$.
- c) $v = f\lambda_{rod}$, $\delta\lambda_{rod} / \lambda_{rod} = (-)\delta f / f \Rightarrow \delta l / l$.
 $\delta l = l \cdot (\delta f / f)$.
 [0.6]
 $\delta l = 1 \times (5.0 \times 10^{-3} / 1.3 \times 10^3) = 3.8 \times 10^{-6} \text{ m}$.
- d) Change in gravitational force on rod at a distance x from the free end $= m\Delta g$ and $m = \rho x A$,
 where A is the cross-sectional area of the rod.
 Change in stress $= m\Delta g / A = \rho x \Delta g$.
 Change in strain $= \delta(dx)/dx = \rho x \Delta g / E$;
 that is, $dx \rightarrow (1 + \rho x \Delta g / E) dx \Rightarrow \Delta l = (\rho \Delta g / 2E) l^2$.
- e) At fundamental $\lambda_{rod} = 4l \Rightarrow \Delta l = \Delta \lambda_{rod} / 4$,
 for $\Delta \lambda_{rod} = 656 \text{ nm} / 10^4 \Rightarrow \Delta l = 656 \text{ nm} / (4 \times 10^4)$.
 $\Delta l = 656 \text{ nm} / (4 \times 10^4) = (\rho \Delta g / 2E) l^2$
 [0.1]
 $\Delta l = (2700 \times 10^{-19} / 14 \times 10^{10}) l^2 \Rightarrow l = 9.2 \times 10^7 \text{ m}$.
- B**
- a) $mc^2 = hf \Rightarrow m = hf / c^2$,
 [0.3]
 $hf' = hf - GMm/R$,
 $\Rightarrow hf' = hf(1 - GM/Rc^2)$, $\therefore f' = f(1 - GM/Rc^2)$.
- b) $n_r = c / c(1 - GM/rc^2)^2$,
 $n_r = 1 + 2GM/rc^2$, for small GM/rc^2 ; i.e. $\alpha = 2$.

c)



Diagram

By Snell's law: $n(r + \delta r) \sin \theta = n(r) \sin (\theta - \delta \xi)$,

$$(n(r) + (dn/dr) \delta r) \sin \theta = n(r) \sin \theta - n(r) \cos \theta \delta \xi.$$

$$(dn/dr) \delta r \sin \theta = -n(r) \cos \theta \delta \xi.$$

Now $n(r) = 1 + 2GM/rc^2$, so $(dn/dr) = -2GM/c^2 r^2$,

$$\text{and } (2GM/c^2 r^2) \sin \theta \delta r = n(r) \cos \theta \delta \xi.$$

$$\text{Hence } \delta \xi = (2GM/c^2 r^2) \tan \theta (\delta r/n) \approx (2GM \tan \theta /c^2 r^2) \delta r.$$

Now $r^2 = x^2 + R^2$, so $r dr = x dx$.

$$\int d\xi = \frac{2GM}{c^2} \int \frac{\tan \theta dr}{r^2} = \frac{2GM}{c^2} \int \frac{\tan \theta r dr}{r^3} = \frac{2GMR}{c^2} \int_{-\infty}^{\infty} \frac{dx}{(x^2 + R^2)^{3/2}}$$

$$\xi = \frac{4GM}{Rc^2} \text{ radians} = 8.4 \times 10^{-6} \text{ radians}.$$